DC-DC converter: Optimisation of the inductor

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Welcome

- *«Most of the basic ideas of the science are essentially simple and can, generally, be expressed in the language which everybody understands»*

  Einstein

Back to Basics

Magnetics is arguably the 'holy grail' of Power Conversion. Understanding it well leads to more optimized converter designs (smaller, cooler…)

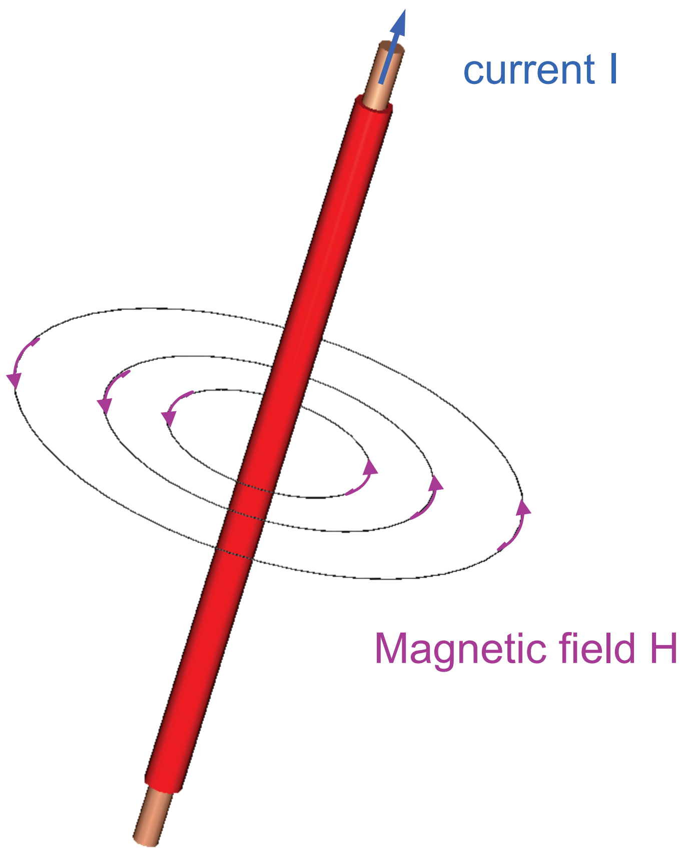
Agenda

- Review of basic knowledge
- Core material
- Skin and proximity effects
- A walk through an example : Buck converter

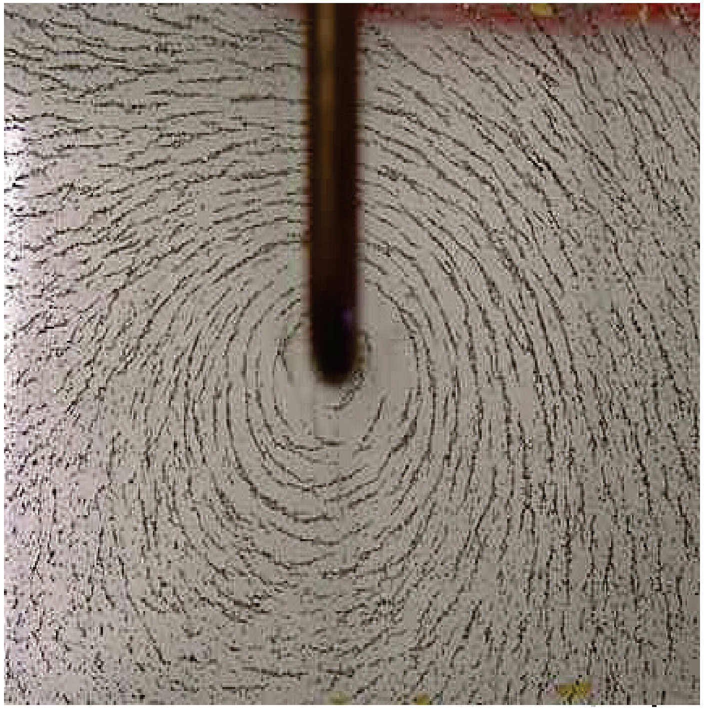
CORE MATERIALS

The magnetic field

Each electric powered wire generates a magnetic field

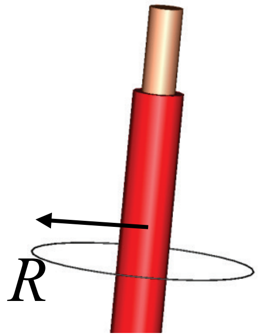


Field model



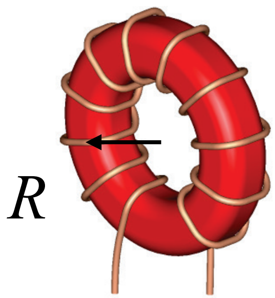
The magnetic field and flux density

Straight wire



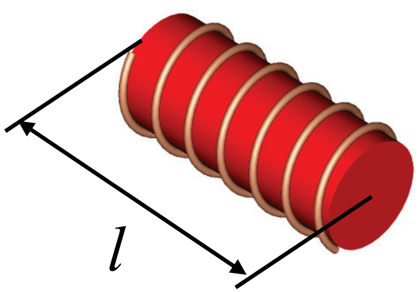
$$H = \frac{I}{2 \cdot \pi \cdot R}$$

Toroidal



$$H = \frac{N \cdot I}{2 \cdot \pi \cdot R}$$

solenoid



$$H = \frac{N \cdot I}{l}$$

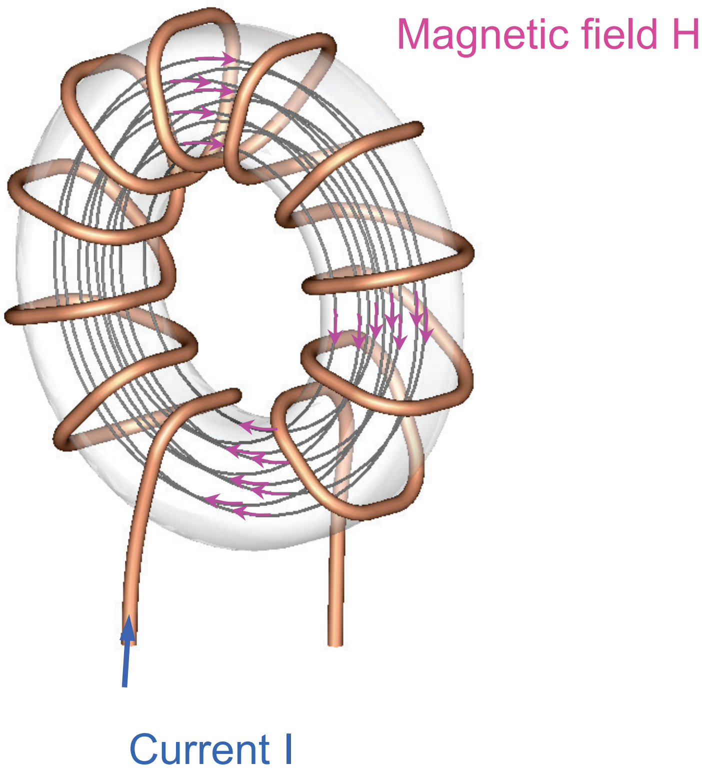
The magnetic field strength is dependent on:

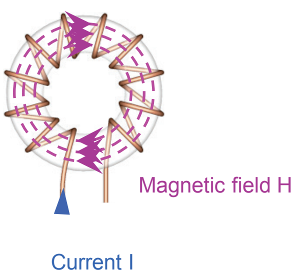
- geometries
- number of turns
- current

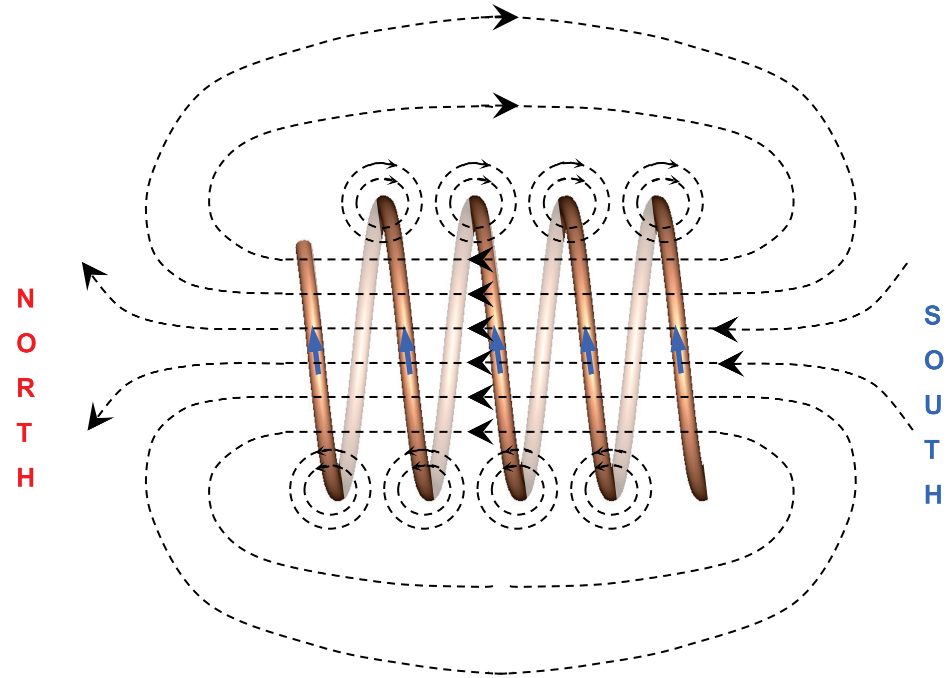
but

NOT ON MATERIAL

The magnetic field







Magnetic Fields – The magnetic field

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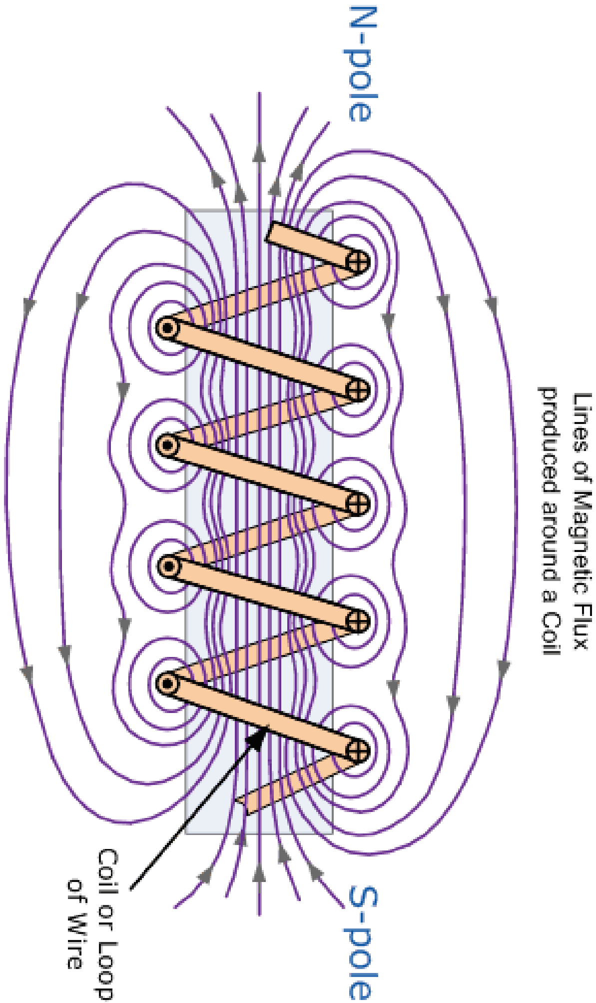
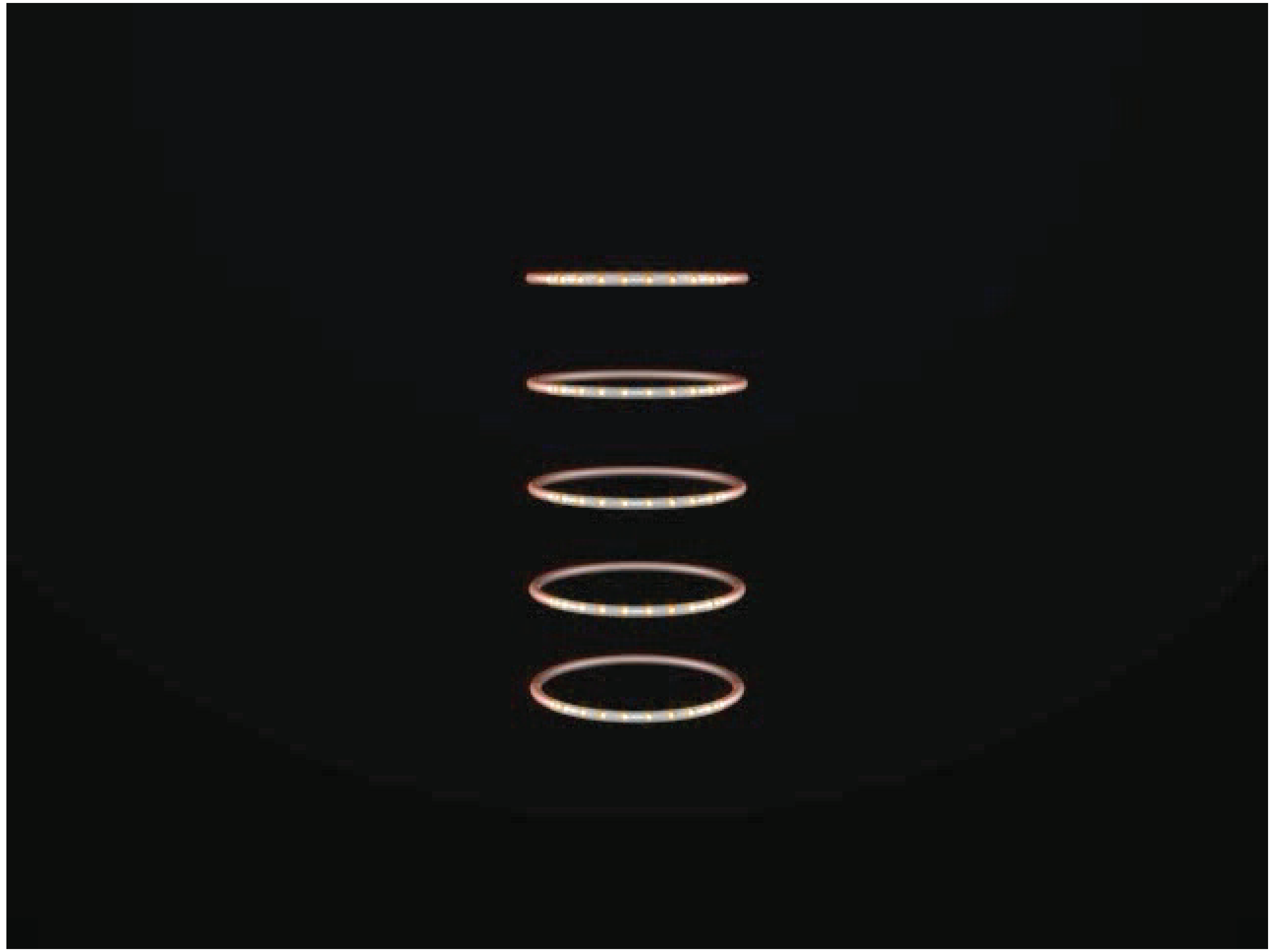
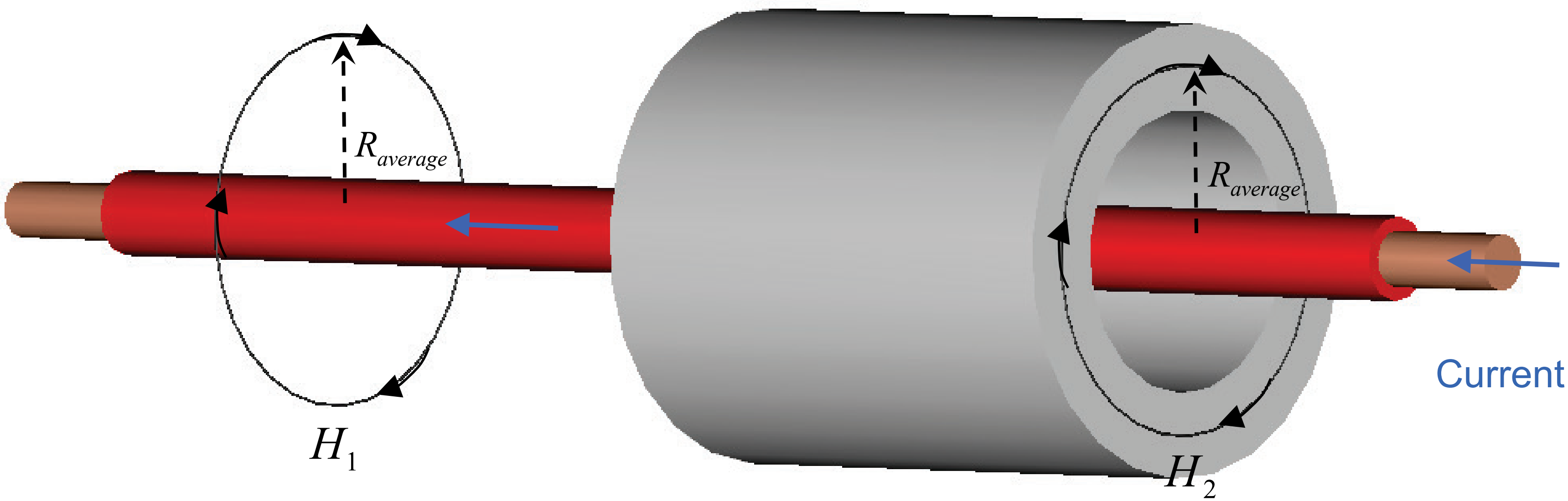




The magnetic field

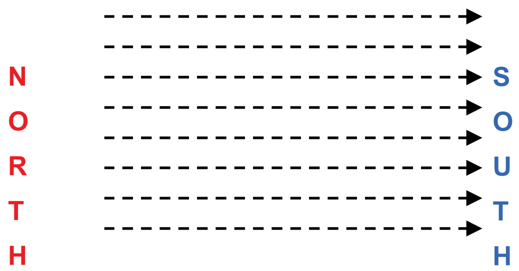


$$H_1 = H_2 = H = \frac{I}{2 \cdot \pi \cdot R_{average}}$$

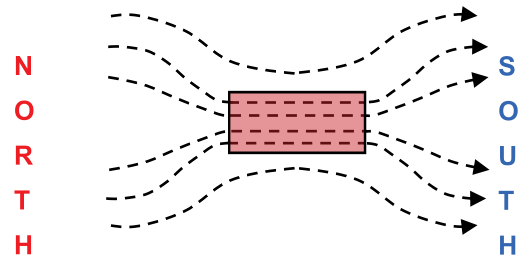
$$B_1 \; \begin{matrix} \neq \\ ? \\ = \end{matrix} \; B_2$$

The magnetic field

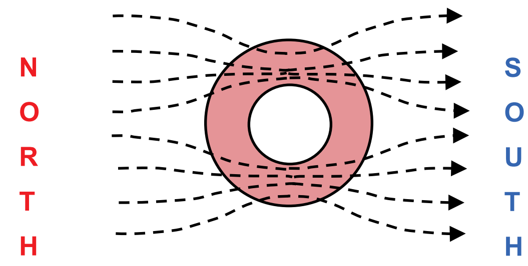
Air



Rod core ferrite



Ring core ferrite



Induction in air:

$$B = \mu_0 \cdot H$$

linear function, because μr = 1 = constant!

Induction in a ferrite:

$$B = \mu_0 \cdot \boxed{\mu_r} \cdot H$$

The relative permeability is a:

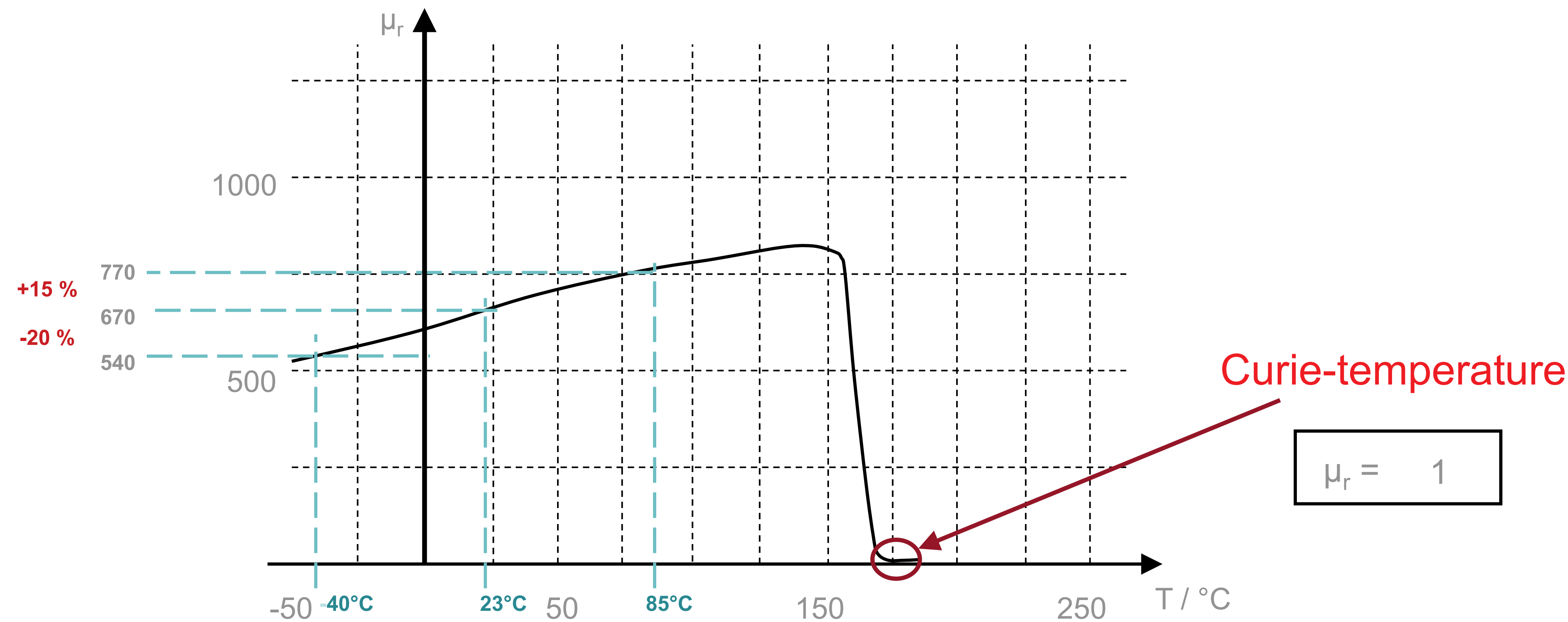
material-
frequency-
temperature-
current-
pressure-

-dependent parameter

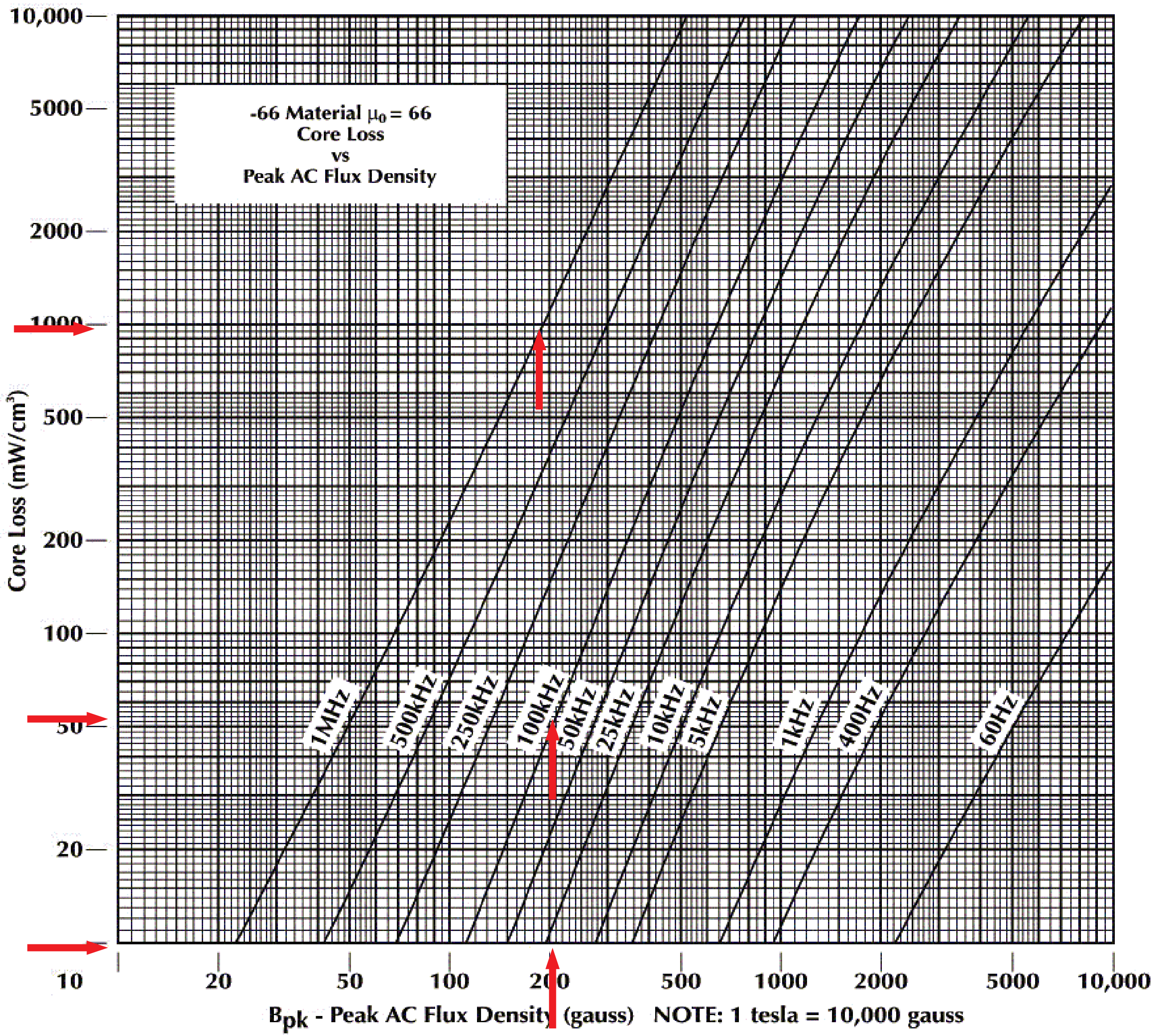
Permeability – core material parameter

Temperature influence

- Relative permeability is dependent on temperature



-66 Material - Core Loss vs Peak AC Flux Density



Effective Permeability vs. Frequency

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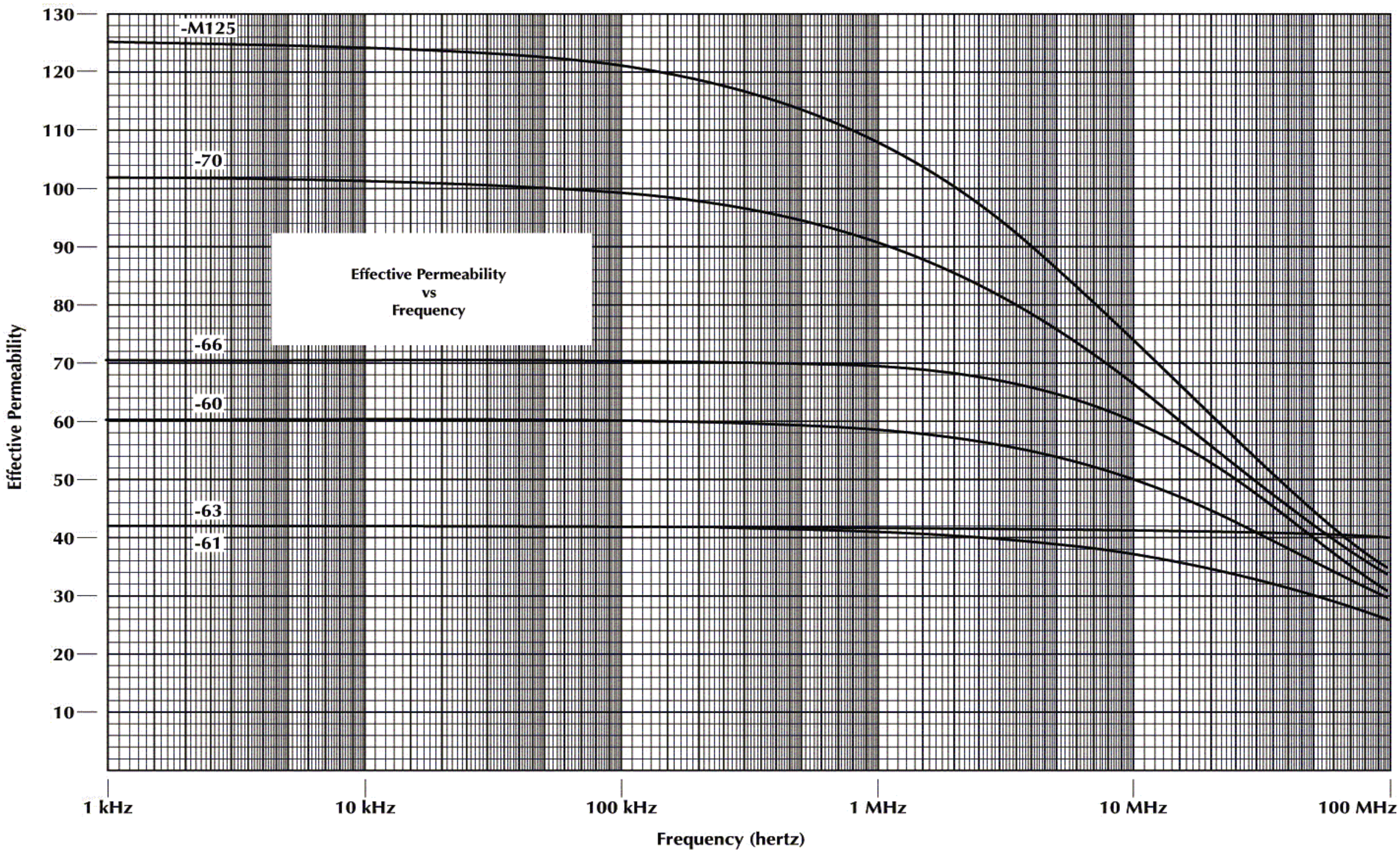

Permeability vs. DC Magnetizing Force

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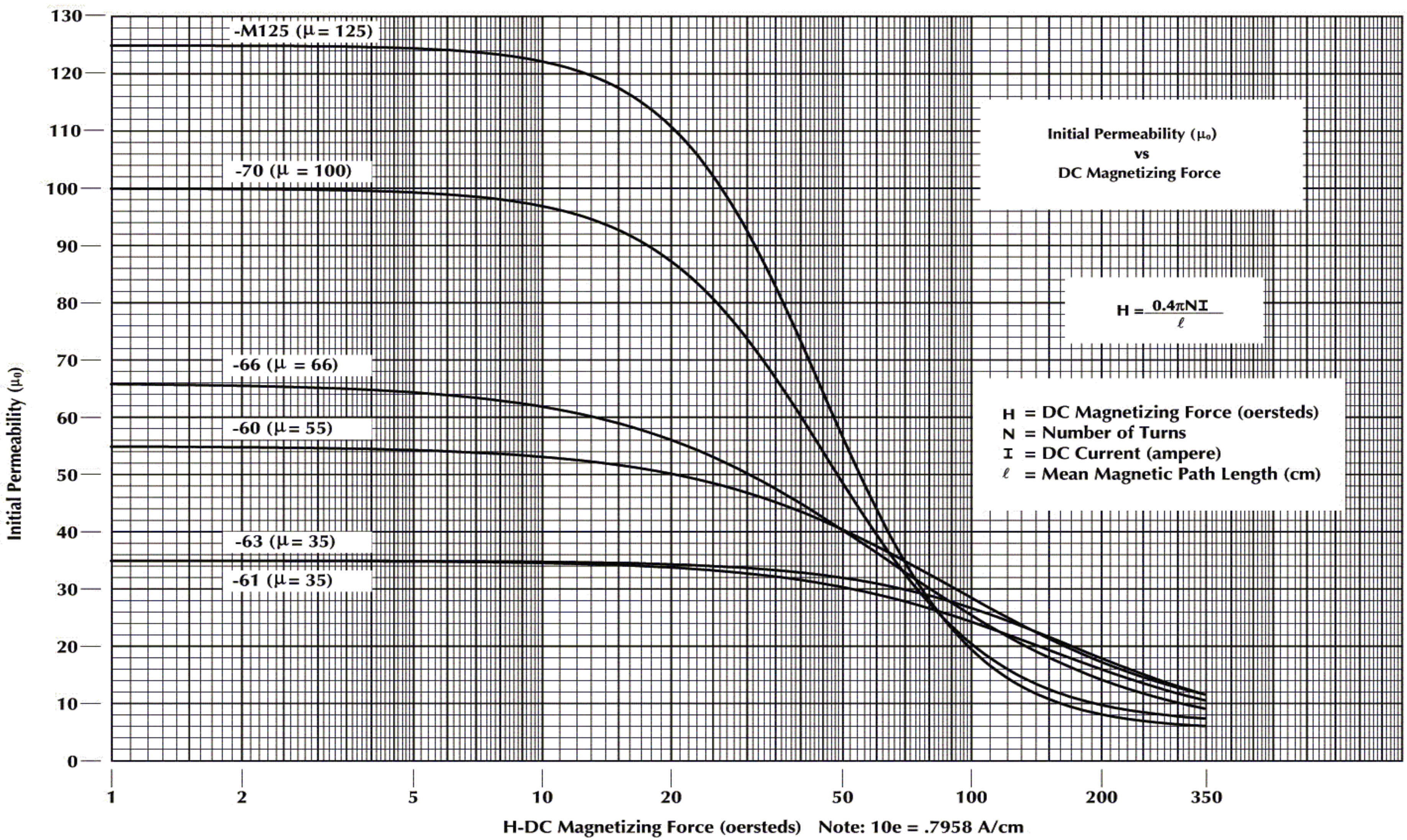

One step beyond

Basic principles

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- <u>1.1 Basic magnetics approach</u>

Materials can be classified by their response to external applied magnetic fields (that is following their magnetics susceptibility)

- In free space

The relationship between B and H is given by: $\boxed{\vec{B} = \mu_0 \vec{H}}$

μ0 magnetic permeability of vaccum

- In material

We define a vector quantity so-called magnetization J given by: $\boxed{\vec{J} = \chi \vec{H}}$

X magnetic susceptibility

We have now a new relationship:

$$\boxed{\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{J}}$$

μ absolute permeability
μr relative permeability

$$\boxed{\vec{B} = \mu_0 (1+\chi) \vec{H} = \mu_0 \mu_r \vec{H} = \mu \vec{H}}$$

Ferromagnetics materials

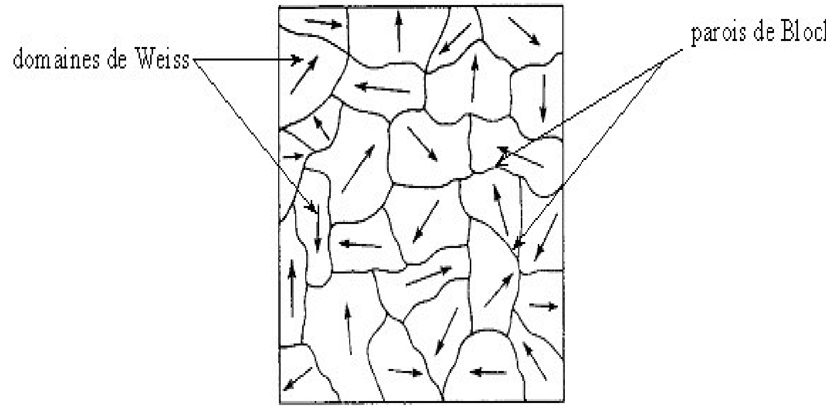
- Ferromagnetic

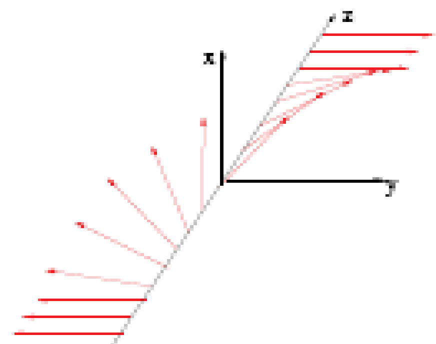
  In ferromagnetics materials coexist differents interactions (as exchange energy…), and in this case nature is « *lasy* » she always favors a state of minimum energy.

- Domaine structure

  A domain (so called Weiss domain) is a region of magnetic material having only one direction of magnetization. Each domain is separated by a Block wall.

  A Bloch wall **is a narrow transition region at the boundary between magnetic domain, over which the magnetization changes from its value in one domain to that in the next**





Hysteresis loop

- Hysteresis loop ***(the history of the magnet)***

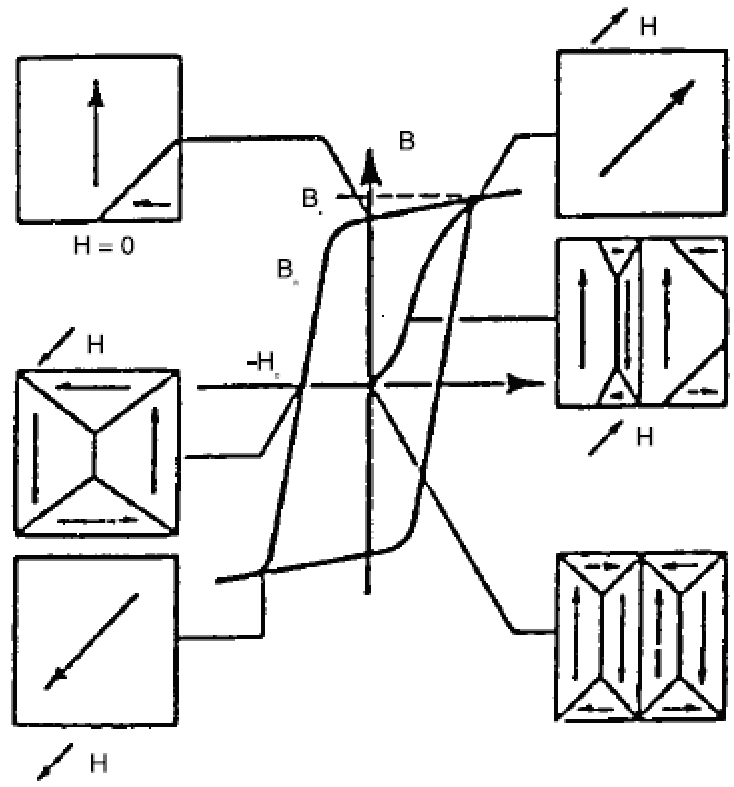
- **Hysteresis results from the shifting of domain walls.**
- **When the magnetisation starts, the domains reorient themselves**

- **Imperfections in the structure obstruct domain wall movement**

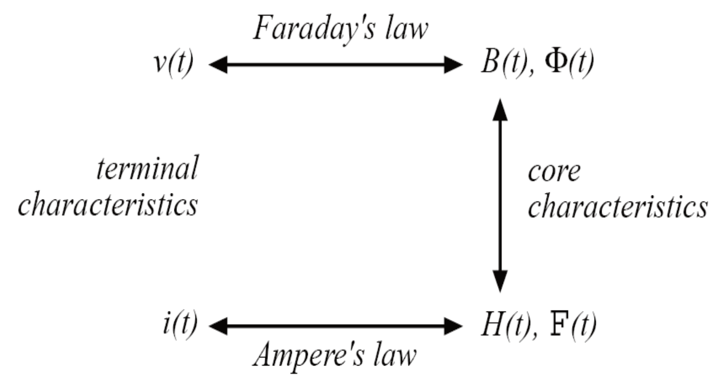
  Walls pass imperfection when the gain in energy is significantly large.

  they hinder the walls from returning to the original state,

  => varying paths upon magnetization and demagnetization.



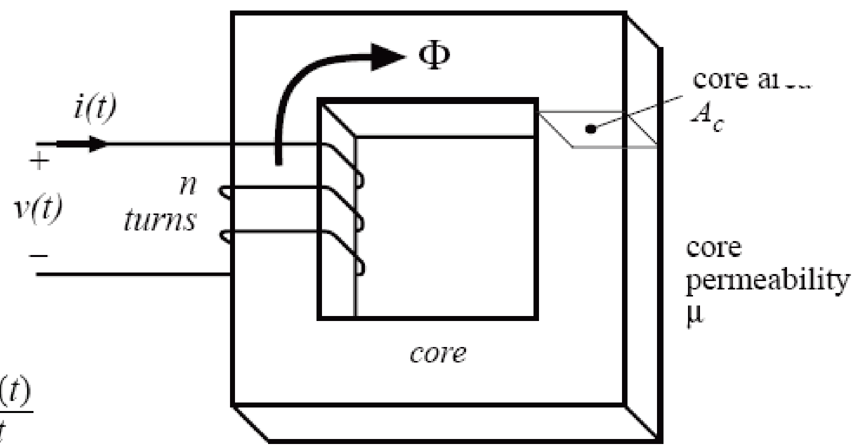
One step Beyond



Faraday's law:

For each turn of wire, we can write

$$v_{turn}(t) = \frac{d\Phi(t)}{dt}$$



Total winding voltage is

$$v(t) = n\, v_{turn}(t) = n\, \frac{d\Phi(t)}{dt}$$

Express in terms of the average flux density $B(t) = \Phi(t)/A_c$

$$v(t) = n\, A_c\, \frac{dB(t)}{dt}$$

Final result

We have:

$$v(t) = n\,A_c \frac{dB(t)}{dt} \qquad H(t)\,l_m = n\,i(t) \qquad B = \begin{cases} B_{sat} & \text{for } H \geq B_{sat}/\mu \\ \mu\,H & \text{for } |H| < B_{sat}/\mu \\ -B_{sat} & \text{for } H \leq B_{sat}/\mu \end{cases}$$

Eliminate B and H, and solve for relation between v and i. For $|\,i\,| < I_{sat}$,

$$v(t) = \mu\,n\,A_c \frac{dH(t)}{dt} \longrightarrow v(t) = \frac{\mu\,n^2 A_c}{l_m} \frac{di(t)}{dt}$$

which is of the form

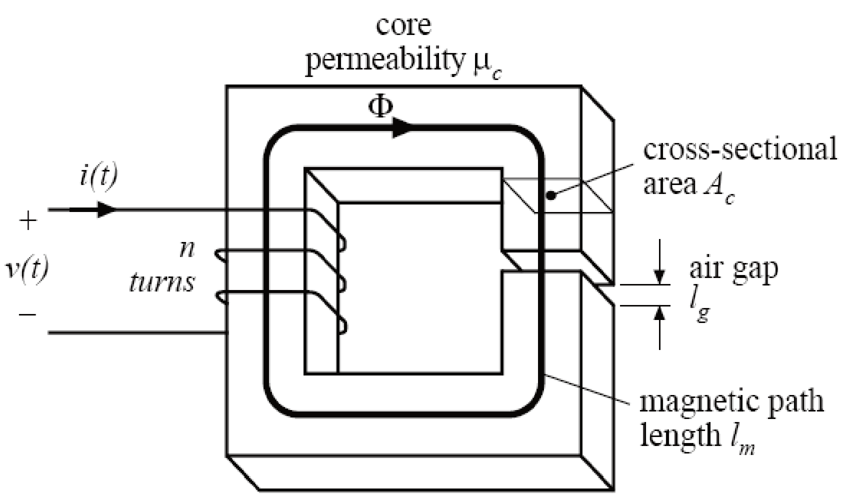
$$v(t) = L \frac{di(t)}{dt} \quad \text{with} \quad L = \frac{\mu\,n^2 A_c}{l_m}$$

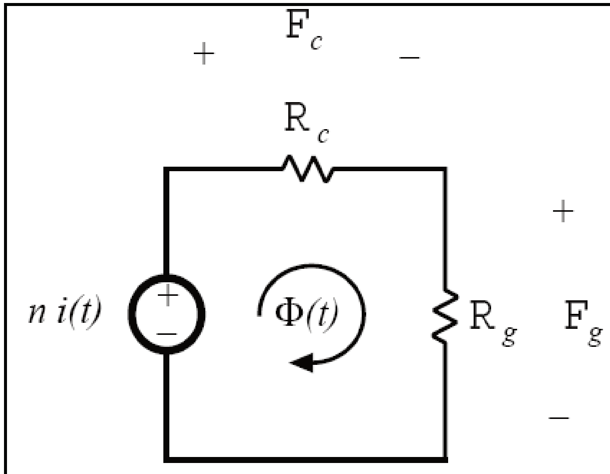
R= Reluctance of core given by: $R = \frac{l_m}{\mu\,A_c}$

=> **L=n²/R**

We can also write: AL = 1/R L=n²AL

Adding an air gap





$$n\,i = \Phi\left(R_c + R_g\right)$$

$$R_c = \frac{l_c}{\mu\,A_c}$$

$$R_g = \frac{l_g}{\mu_0\,A_c}$$

Faraday's law: $$v(t) = n\,\frac{d\Phi(t)}{dt}$$

Substitute for Φ: $$v(t) = \frac{n^2}{R_c + R_g}\,\frac{di(t)}{dt}$$

Hence inductance is

$$L = \frac{n^2}{R_c + R_g}$$

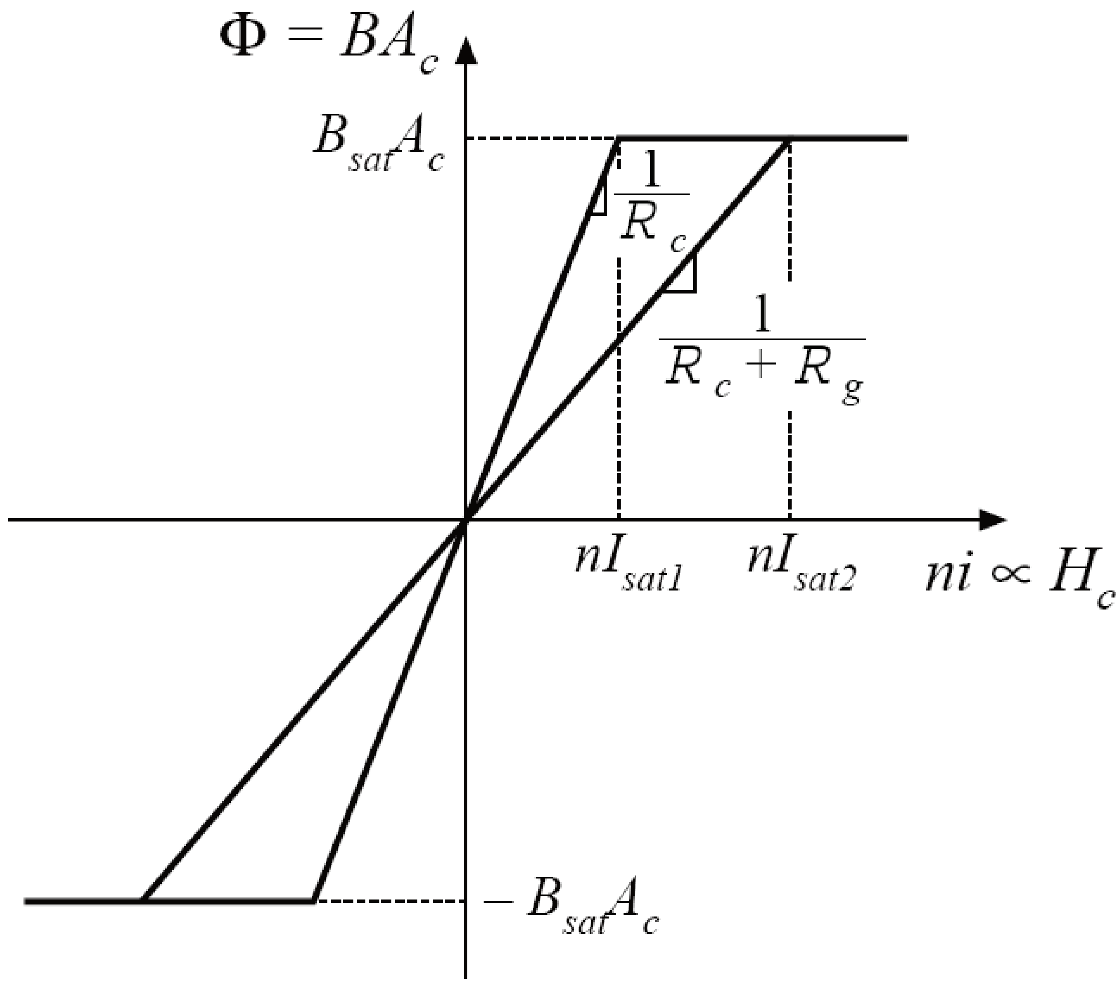
B-H-Curve with airgap

$$n\,i = \Phi\left(R_c + R_g\right)$$

$$L = \frac{n^2}{R_c + R_g}$$

$$\Phi_{sat} = B_{sat}A_c$$

$$I_{sat} = \frac{B_{sat}A_c}{n}\left(R_c + R_g\right)$$



Effect of air gap:

- decrease inductance
- increase saturation current
- inductance is less dependent on core permeability

Sumarize part 1

$$\boxed{V = N\frac{d\phi}{dt} = NA\frac{dB}{dt} = L\frac{dI}{dt}}$$

These are all forms of the ‘voltage-dependent equation’, called Faraday’s Law.

We can also eliminate the voltage from the last two to get the ‘voltage-independent equation’ below

$$\boxed{\Delta B = \frac{L\Delta I}{NA}}$$

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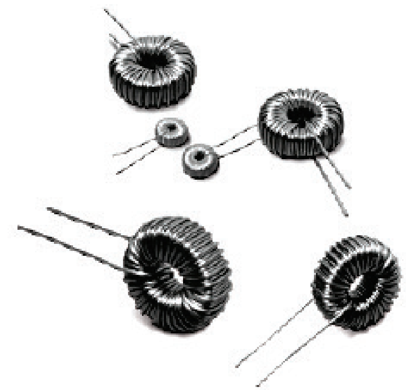
Inductance and <u>Size</u> of Inductor

- **Reducing Inductance does not *necessarily* mean we are reducing physical <u>size</u> of inductor!**

<u>Remember:</u>



- **'Inductance ≠ Inductor'**

- **We can create almost any inductance on almost any size of core by placing required number of turns on it!**

Losses

losses generally

$$P_{total} = P_{CORE} + P_{Cu}$$

core losses

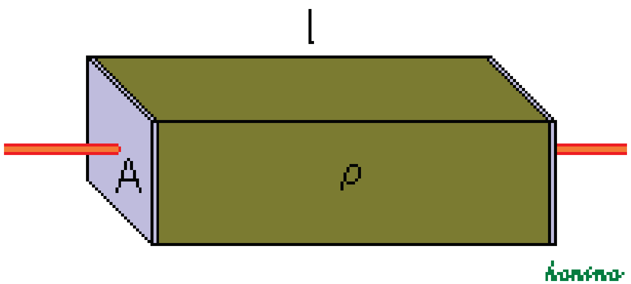
- Hysteresis losses
- Eddy current losses

copper losses

- DC losses – depending on DCR
- AC-losses – dep. on winding structure
  - → Skin-Effect
  - → Proximity-Effect

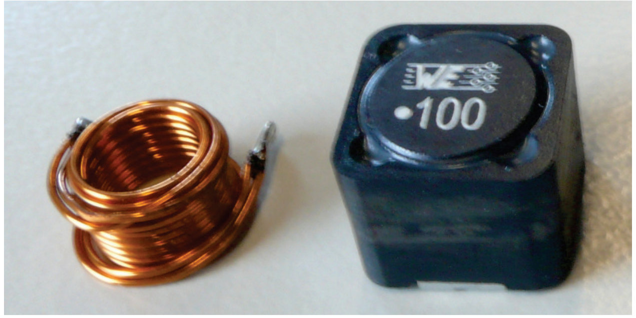
The target is to reduce as much as possible these losses

Copper Loss – DC



$$DCR = \rho \cdot \frac{l}{A}$$

$$\rho = 1{,}786 \cdot 10^{-2}\ \frac{\Omega \cdot mm^2}{m}$$



$$P_{cu} = I_{rms}^2 R$$

Eddy Currents Cause Skin and Proximity Effect

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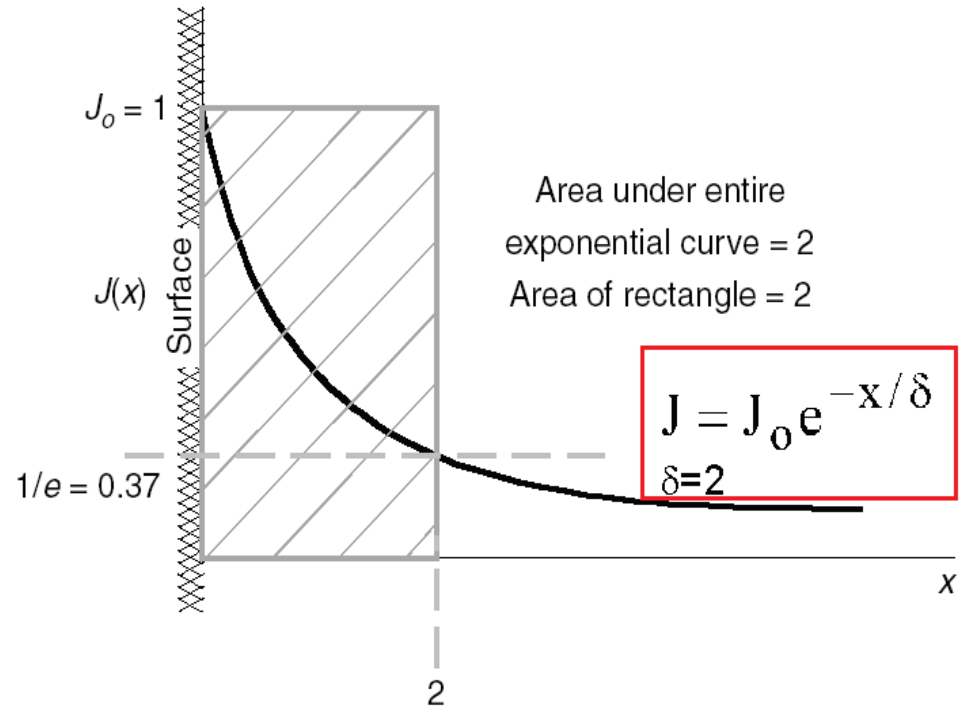
So What are Eddy Currents?

Discovered by Leon Foucault in 1851

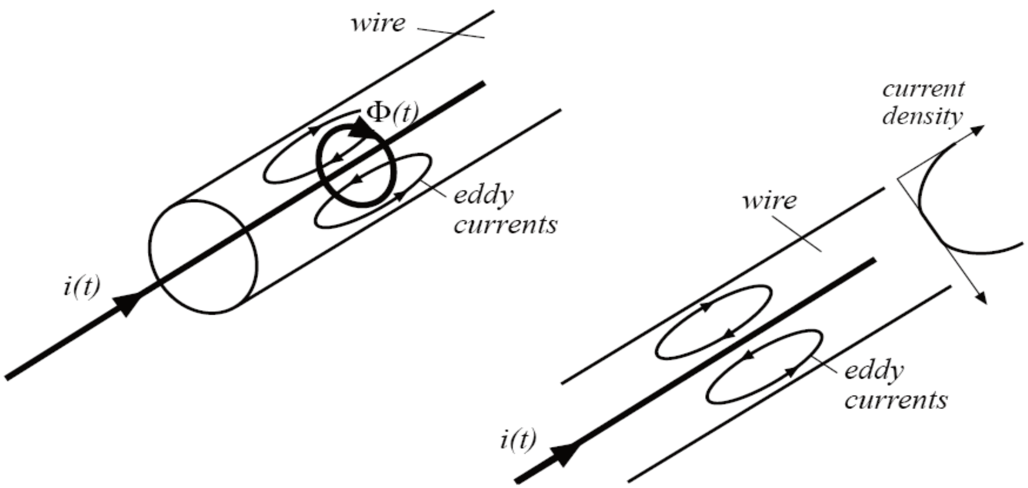
Eddy Currents - Skin Effect

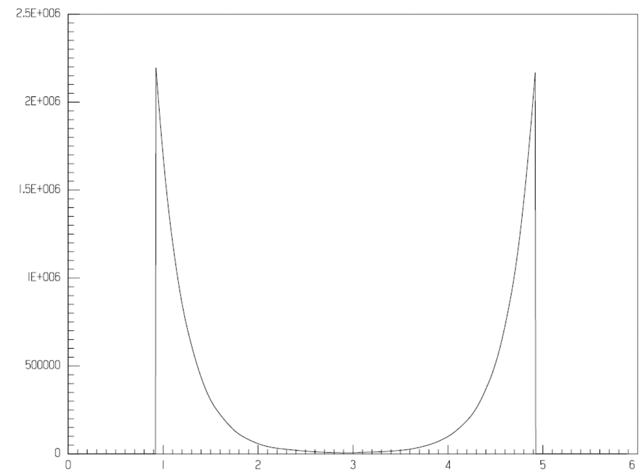
- **Formula for Single Isolated Wire in free Space away from Influence of Other Magnetic Fields.**
- **Penetration Depth is Where Current Density has Dropped to 1/e (Napier's constant or Euler's number) or 36.78% of Surface Density.**

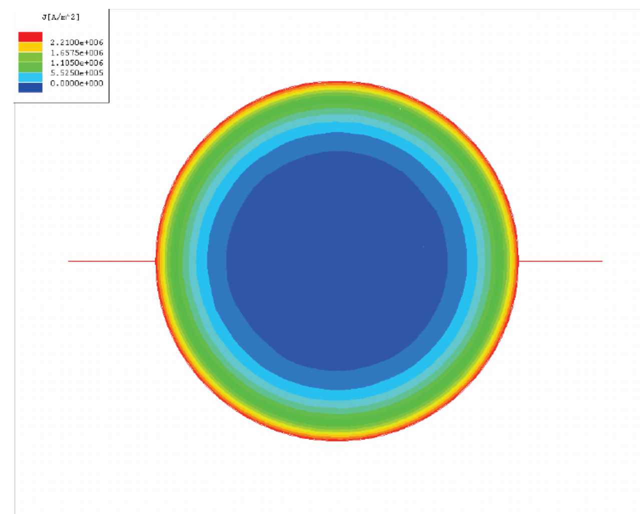


$$\delta = \sqrt{\frac{\rho}{\pi \bullet \mu \bullet f}}$$

Eddy Currents in Wire







Skin depth

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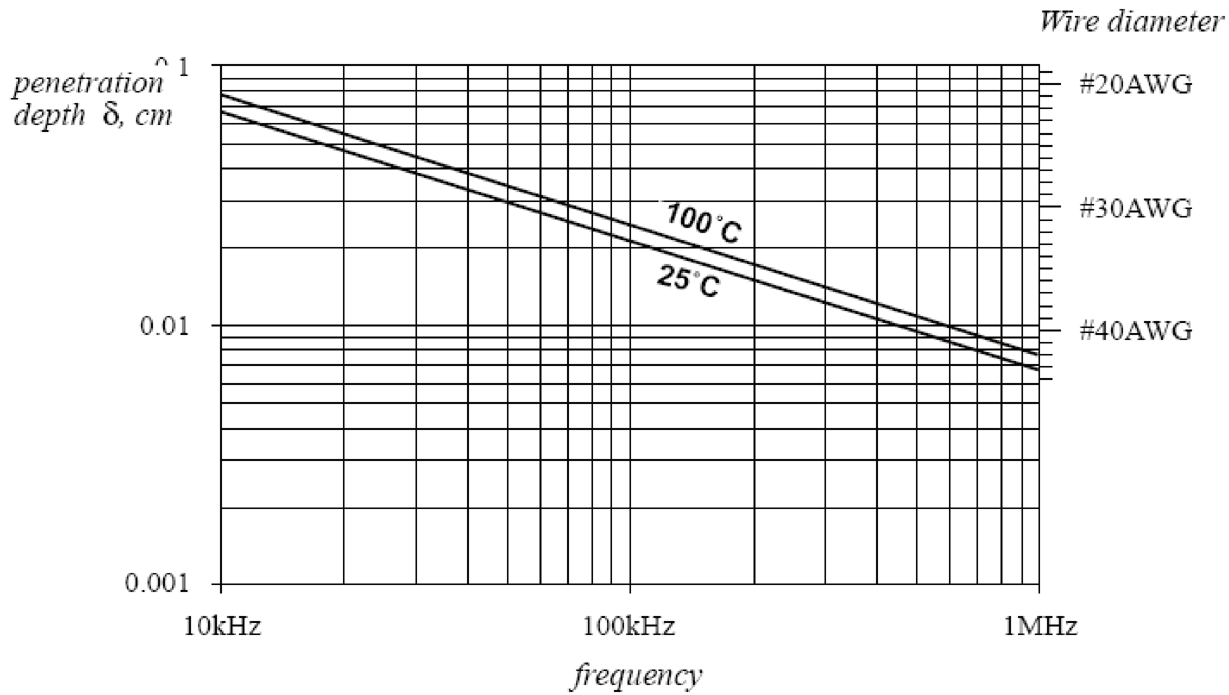
$$\delta = \frac{76}{\sqrt{f}} mm$$

$$\delta = \sqrt{\frac{2 \cdot \rho}{\omega \cdot \mu}}$$



<u>For Copper</u>

δ @ 50Hz = 9.38 mm
δ @ 10kHz = 0.66 mm
δ @ 100kHz = 0.22 mm

δ = skin depth
ρ = resistivity
ω = angular frequency 2πf
μ = permeability

Proximity Effect – Caused by Eddy Currents

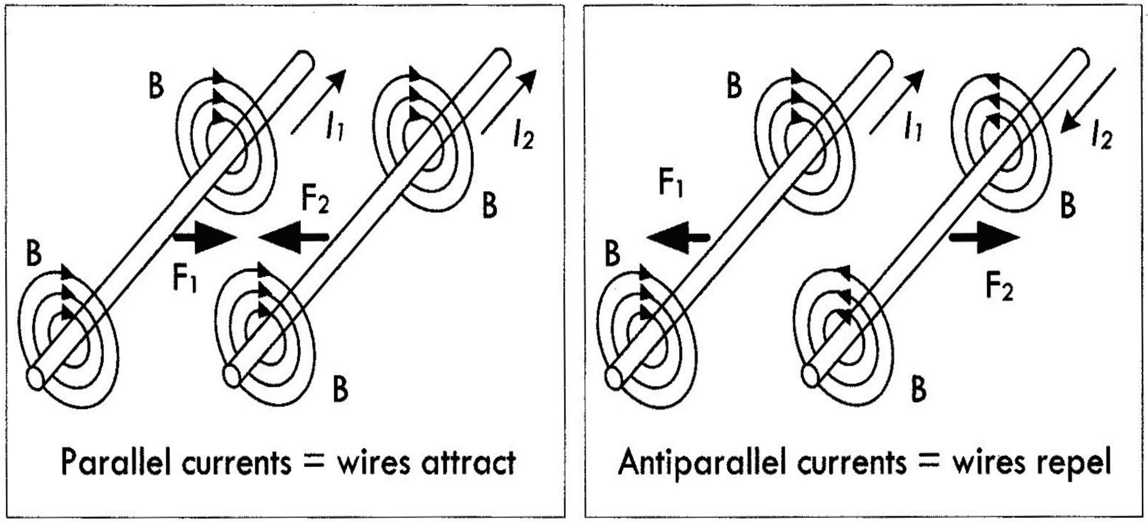
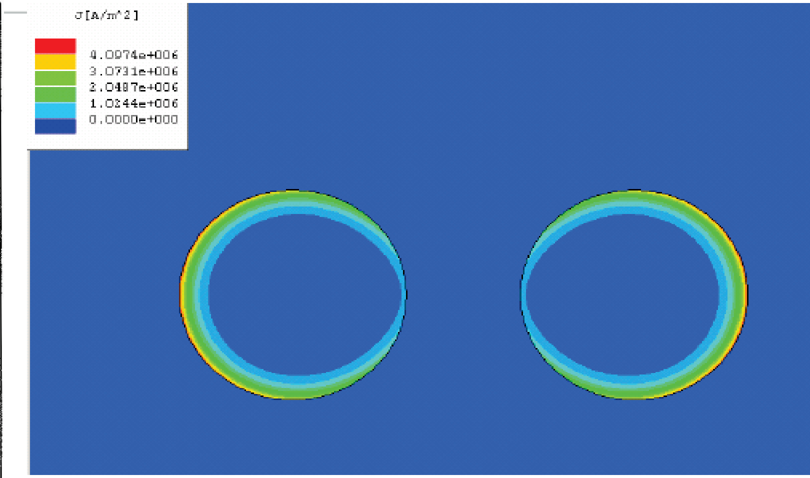
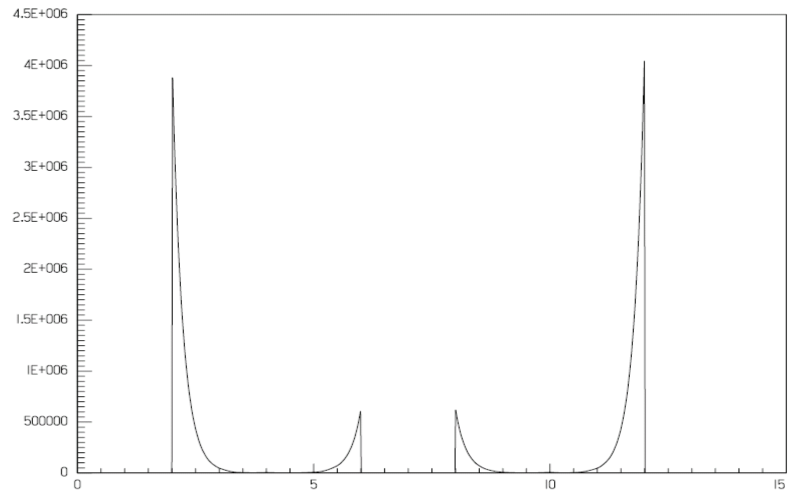


FIGURE 2.114 Forces exerted between two current-carrying wires.

- **Proximity Effect Creates More Loss than Skin Effect**
- **Approximate Calculations**
  - **Dowell's curves for simple geometries**
  - **Work by Bruce Carsten or Charles Sullivan for more in-depth**





Current in same direction

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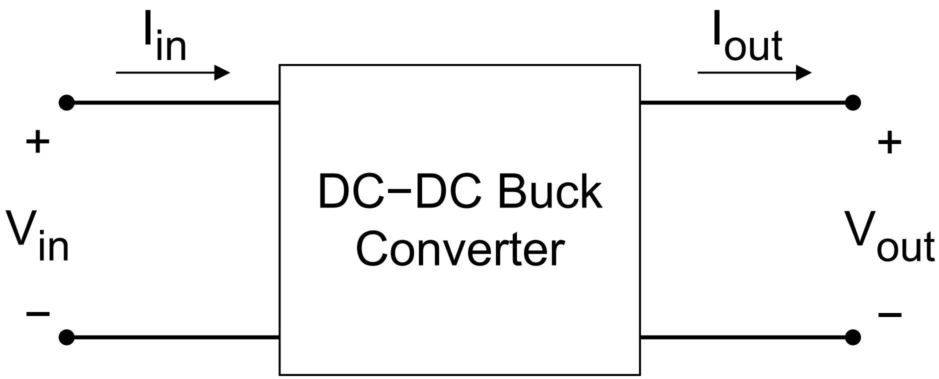
A walk through an example : Buck converter

DC/DC Buck converter : Objective – to efficiently reduce DC voltage

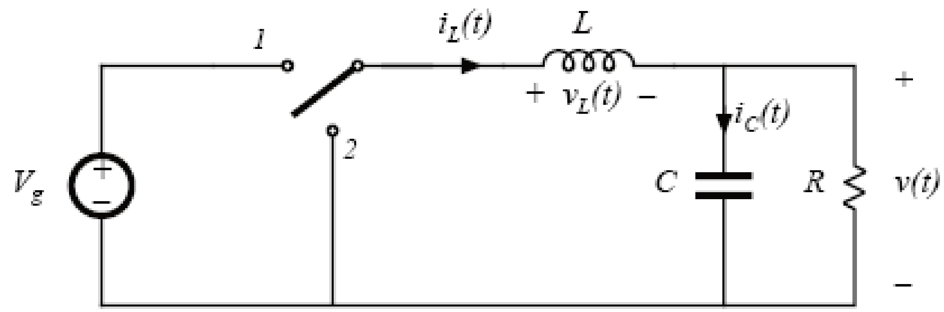


Lossless objective: $P_{in} = P_{out}$, which means that $V_{in}I_{in} = V_{out}I_{out}$ and

$$\frac{V_{out}}{V_{in}} = \frac{I_{in}}{I_{out}}$$

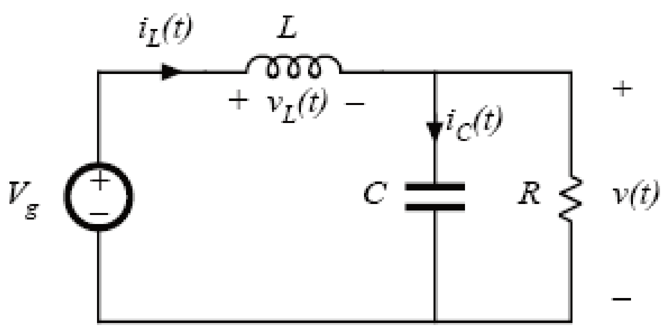
Buck converter analysis

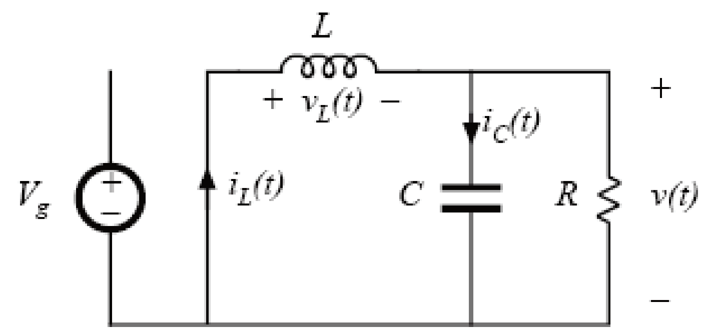
original converter



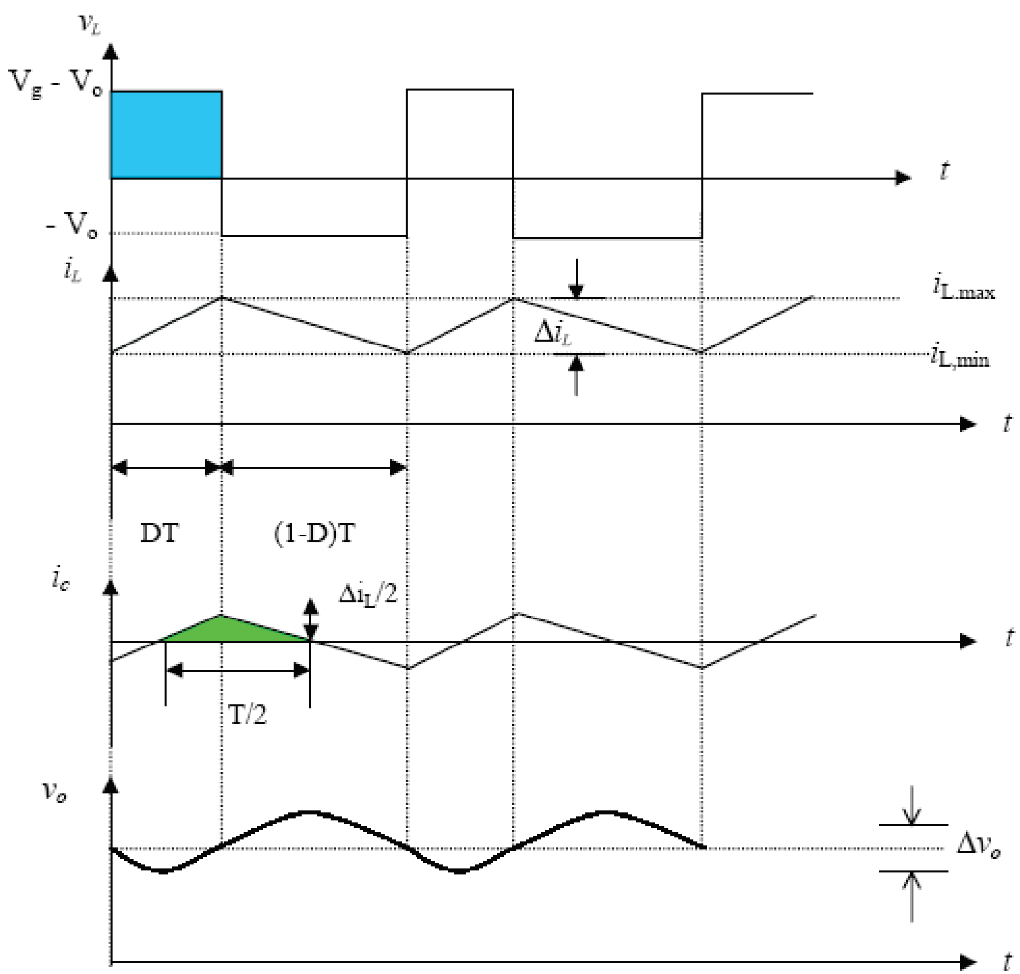
switch in position 1

switch in position 2



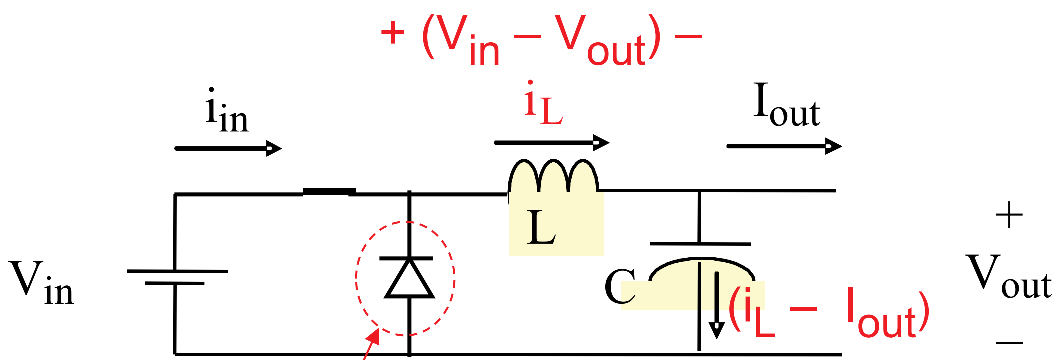


Key waveforms



The input/output equation for DC-DC converters usually comes by examining inductor voltages

Switch closed for DT seconds

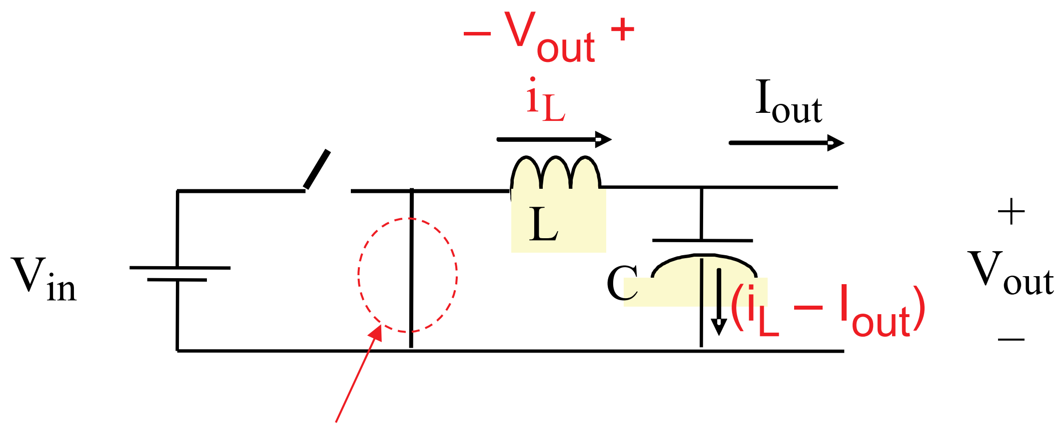


Reverse biased, thus the diode is open

$$v_L = L\frac{di_L}{dt}, \quad \boxed{v_L = V_{in} - V_{out},} \quad V_{in} - V_{out} = L\frac{di_L}{dt}, \quad \frac{di_L}{dt} = \frac{V_{in} - V_{out}}{L}$$

for DT seconds

Switch open for (1 − D)T seconds



i_L continues to flow, thus the diode is closed. This is the assumption of "continuous conduction" in the inductor which is the normal operating condition.

$$v_L = L\frac{di_L}{dt}, \qquad \boxed{v_L = -V_{out},} \qquad -V_{out} = L\frac{di_L}{dt}, \qquad \frac{di_L}{dt} = \frac{-V_{out}}{L}$$

for (1−D)T seconds

Since the average voltage across L is zero

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$$V_{Lavg} = D \bullet (V_{in} - V_{out}) + (1 - D) \bullet (-V_{out}) = 0$$

$$DV_{in} = D \cancel{\bullet} V_{out} + V_{out} - D \cancel{\bullet} V_{out}$$

The input/output equation becomes $\boxed{V_{out} = DV_{in}}$

From power balance, $V_{in}I_{in} = V_{out}I_{out}$ **, so**

$$\boxed{I_{out} = \frac{I_{in}}{D}}$$

Take a closer look on the inductor current (CCM)

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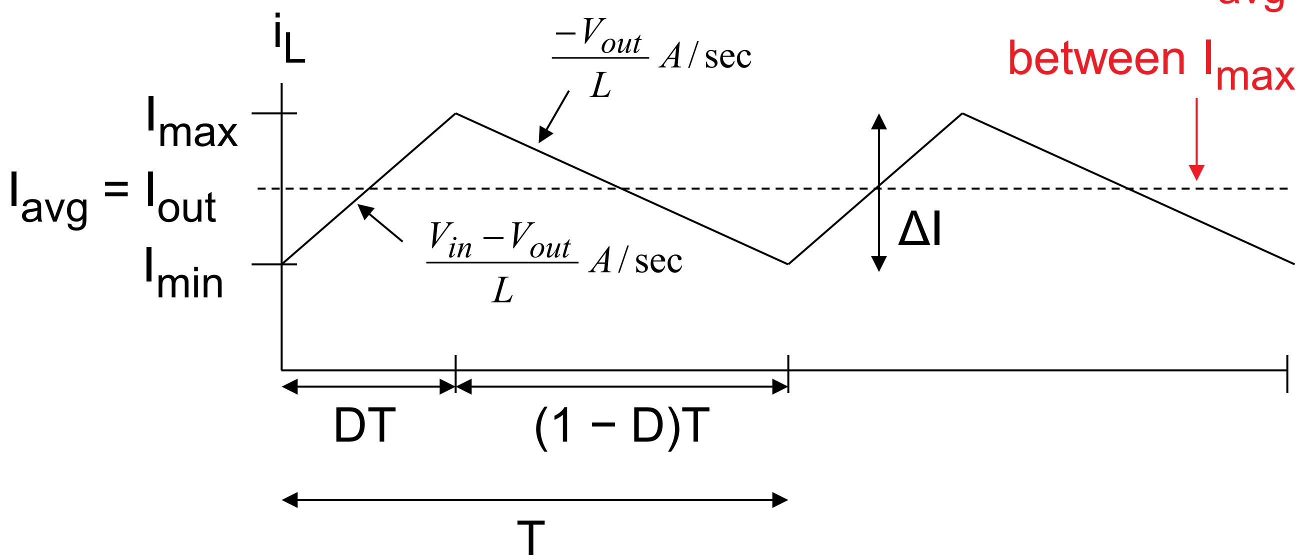
Switch closed, $v_L = V_{in} - V_{out}, \dfrac{di_L}{dt} = \dfrac{V_{in} - V_{out}}{L}$

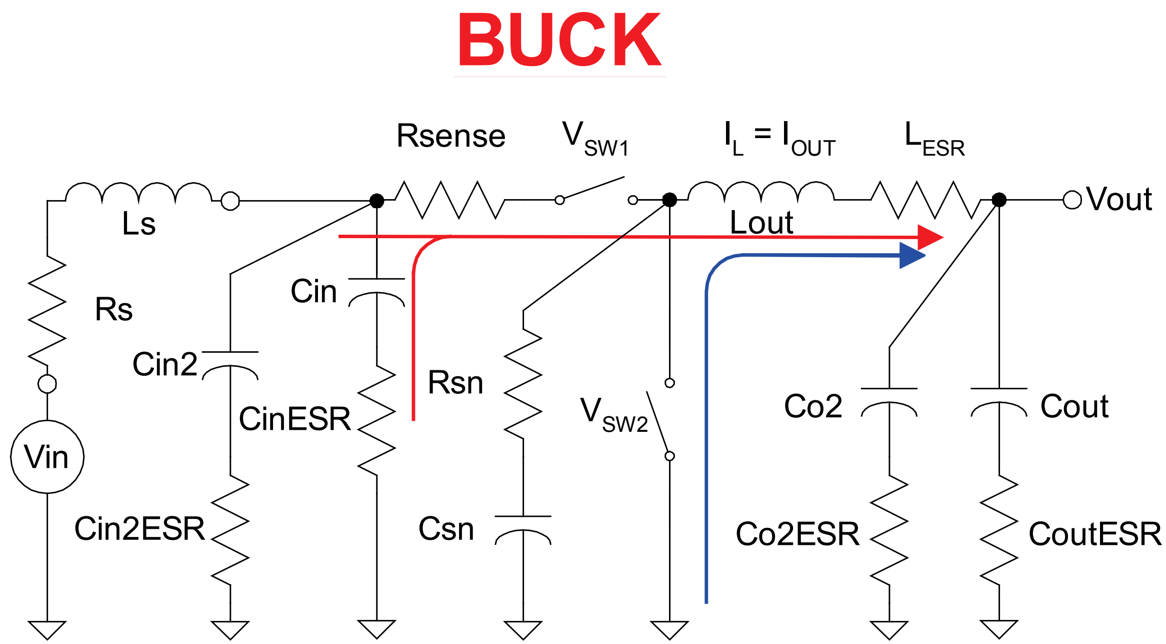
Switch open, $v_L = -V_{out}, \dfrac{di_L}{dt} = \dfrac{-V_{out}}{L}$

From geometry, $I_{avg} = I_{out}$ is halfway between I_{max} and I_{min}

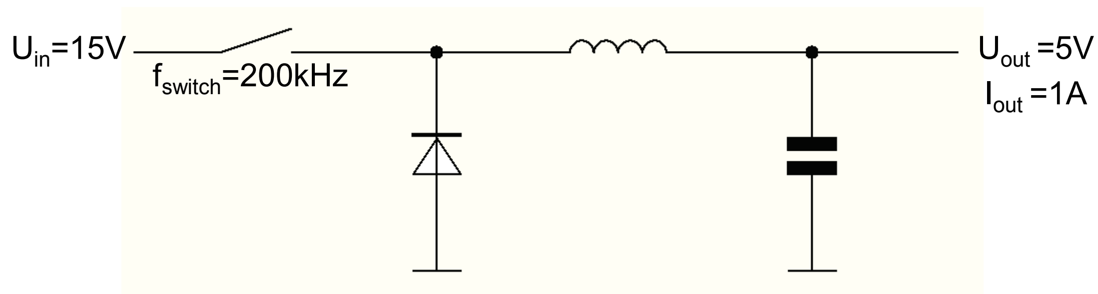


Periodic – finishes a period where it started

Schematic can be more or less complex



Inductor selection : Buck converter



$Vin := 15V$

$Fsw := 200000Hz$

$Vsw := 1V$

$Vout := 5V$

$Vd := 0.6V$

What is the only degree of freedom available to the designer when he is designing this converter?

Ripple current

$$r = \frac{\Delta I}{I_C}$$

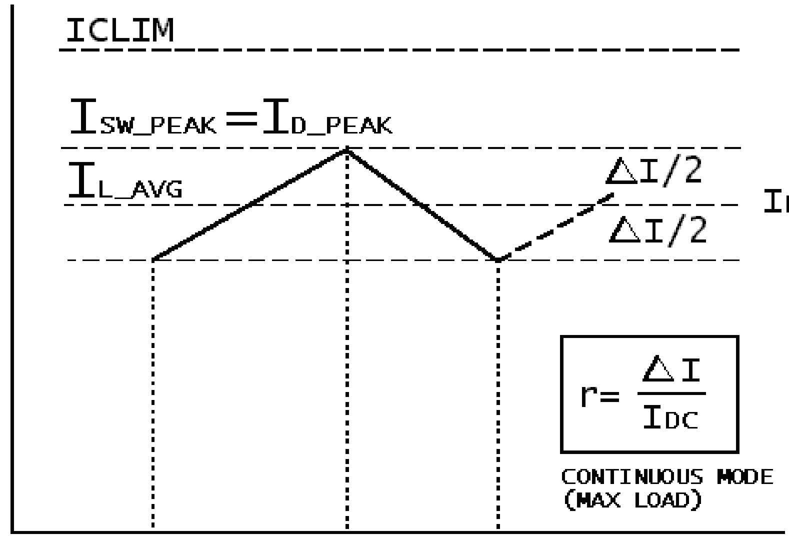
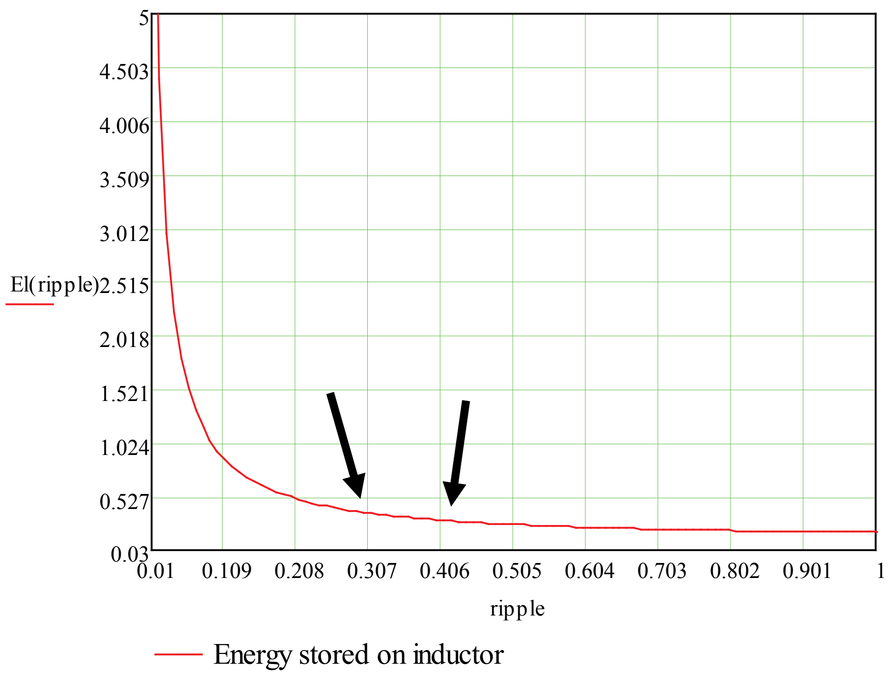


FIGURE 1

- **'r' is by definition related to the average of the <u>inductor current</u> (the geometric center, or the 'DC value', and the total swing in the current ΔI.**

- **Current ripple ratio 'r' connects both the AC and the DC components of the inductor current. It scales with the load current. It is also a 'geometrical' factor <u>that is independent of frequency.</u>**

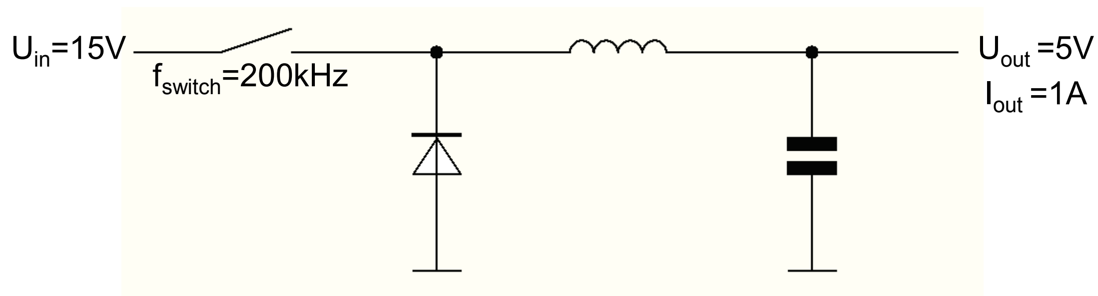
Energy stored into inductor Vs Ripple current



$$Energy := \frac{1}{2} \cdot L \cdot Ipk^2$$

$$Energy := \frac{1}{2} Iout \cdot V\mu sec \cdot \frac{1}{r} \cdot \left(1 + \frac{r}{2}\right)^2$$

Inductor selection : Buck converter



$Vin := 15V$

$Fsw := 200000Hz$

$Vsw := 1V$

$r := 0.4$

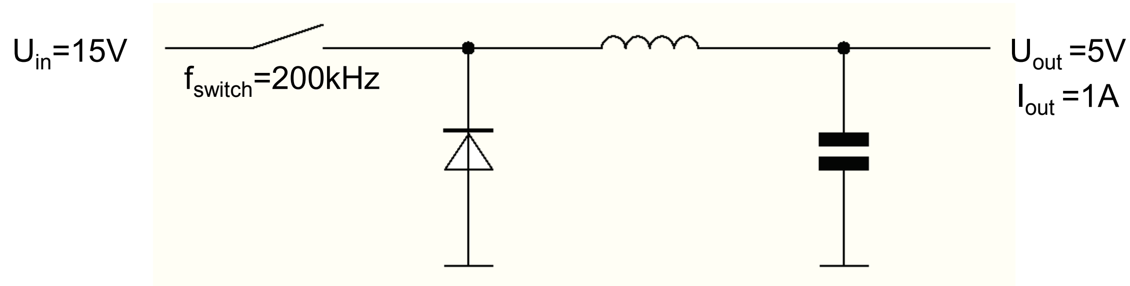
$Vout := 5V$

$Vd := 0.6V$

$$\Delta I := r \cdot Iout = 0.4\,A$$

$$Ipeak := Iout + \frac{\Delta I}{2} = 1.2\,A$$

Inductor selection : Buck converter



Duty cycle

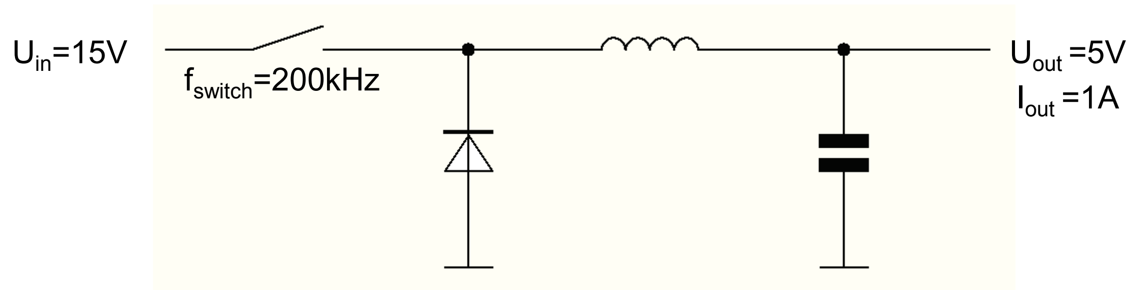
$$D1 := \frac{Vout}{Vin} = 0.333$$

$$D4 := \frac{Vout + Vd}{Vin + Vd} = 0.359$$

$$D2 := \frac{Vout + Vd}{Vin - Vsw + Vd} = 0.384$$

$$D3 := \frac{Vout + Vd + Iout \cdot Rl}{Vin - Vsw + Vd} = 0.391$$

Inductor selection : Buck converter



Switch on time and Volt µsecond law

$$\mathrm{Ton1} := \frac{\mathrm{D1}}{\mathrm{Fsw}} = 1.667 \times 10^{-6}\,\mathrm{s}$$

$$\mathrm{V\mu sec1} := (\mathrm{Vin} - \mathrm{Vout})\cdot \mathrm{Ton1}$$

$$\mathrm{Ton2} := \frac{\mathrm{D2}}{\mathrm{Fsw}} = 1.918 \times 10^{-6}\,\mathrm{s}$$

$$\mathrm{V\mu sec2} := (\mathrm{Vin} - \mathrm{Vsw} - \mathrm{Vout})\cdot \mathrm{Ton2}$$

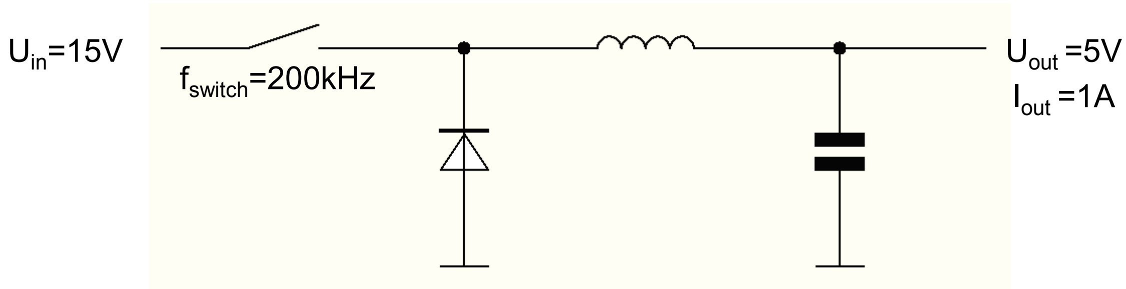
$$\mathrm{Ton4} := \frac{\mathrm{D4}}{\mathrm{Fsw}} = 1.795 \times 10^{-6}\,\mathrm{s}$$

$$\mathrm{V\mu sec4} := (\mathrm{Vin} - \mathrm{Vout})\cdot \mathrm{Ton4}$$

$$\mathrm{Ton3} := \frac{\mathrm{D3}}{\mathrm{Fsw}} = 1.955 \times 10^{-6}\,\mathrm{s}$$

$$\mathrm{V\mu sec3} := (\mathrm{Vin} - \mathrm{Vout} - \mathrm{Vsw} - \mathrm{Rl}\cdot \mathrm{Iout})\cdot \mathrm{Ton3}$$

Inductor selection : Buck converter



$$L1 := \frac{V\mu sec1}{r \cdot Iout} = 4.167 \times 10^{-5} H$$

$$L2 := \frac{V\mu sec2}{r \cdot Iout} = 4.315 \times 10^{-5} H$$

$$L3 := \frac{V\mu sec3}{r \cdot Iout} = 4.346 \times 10^{-5} H$$

$$L4 := \frac{V\mu sec4}{r \cdot Iout} = 4.487 \times 10^{-5} H$$

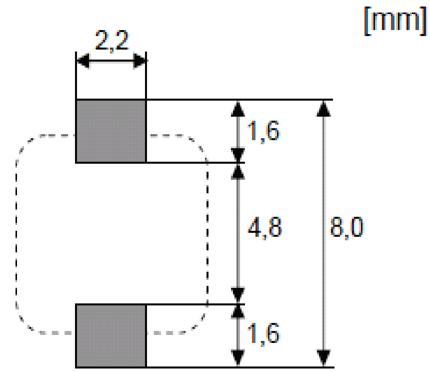
Inductor selection: Lets go with few *mistakes*

First selection: 744777147 (WE-PD type M)

B Elektrische Eigenschaften / electrical properties :

Eigenschaften / properties	Testbedingungen / test conditions		Wert / value	Einheit / unit	tol.
Induktivität / inductance	**1 kHz / 0,25V**	L	**47,0**	µH	**±20%**
DC-Widerstand / DC-resistance	**@ 20°C**	$R_{DC\ typ}$	**0,1700**	Ω	**typ.**
DC-Widerstand / DC-resistance	**@ 20°C**	$R_{DC\ max}$	**0,2600**	Ω	**max.**
Nennstrom / rated current	**ΔT=40 K**	I_{DC}	**1,03**	A	**max.**
Sättigungsstrom / saturation current	**|ΔL/L|<10%**	I_{sat}	**1,10**	A	**typ.**
Eigenres.-Frequenz / self-res.-fequency		SRF	**10,3**	MHz	**typ.**

C Lötpad / soldering spec. :



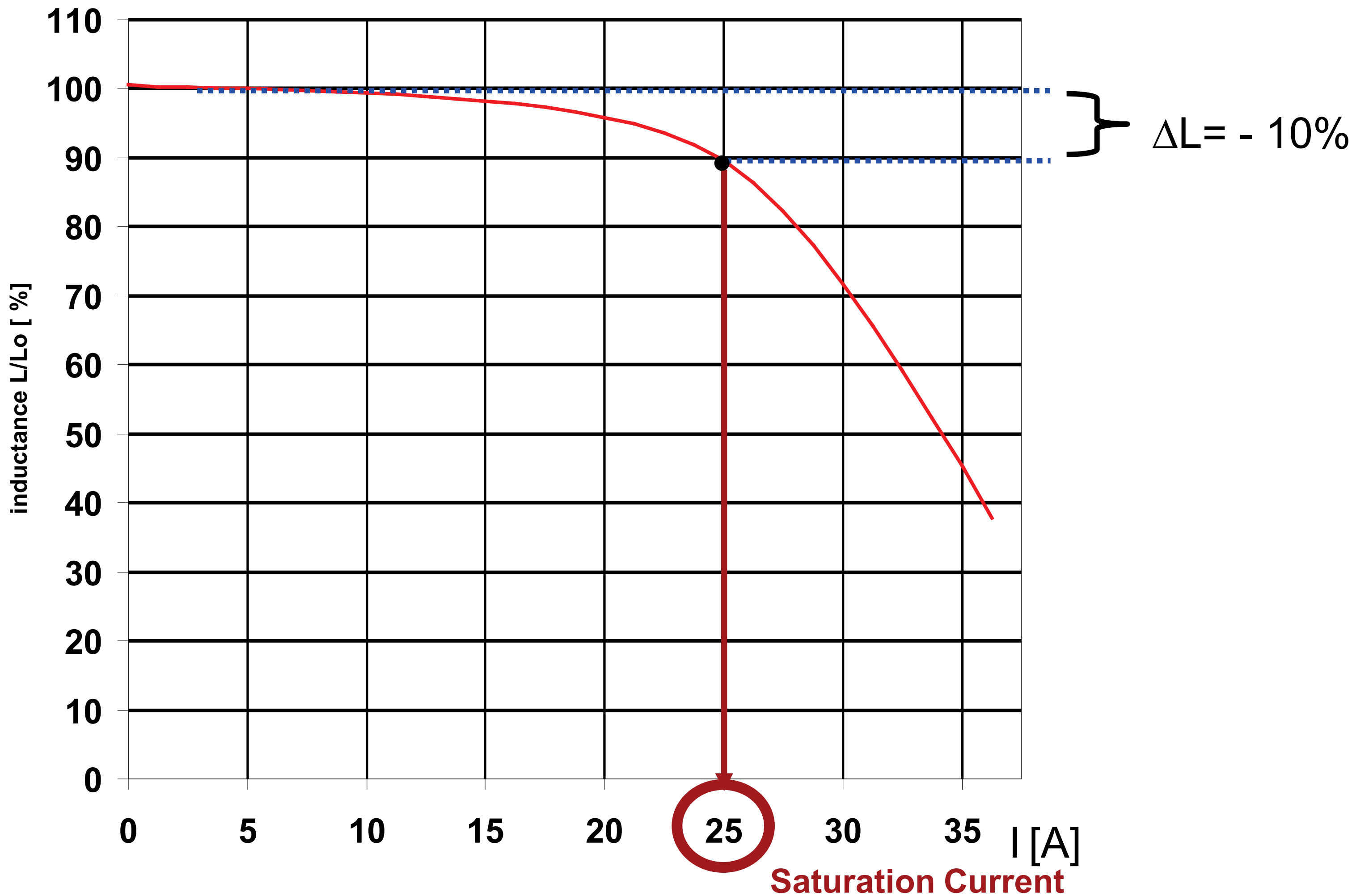
WÜRTH ELEKTRONIK

Saturation current

Definition

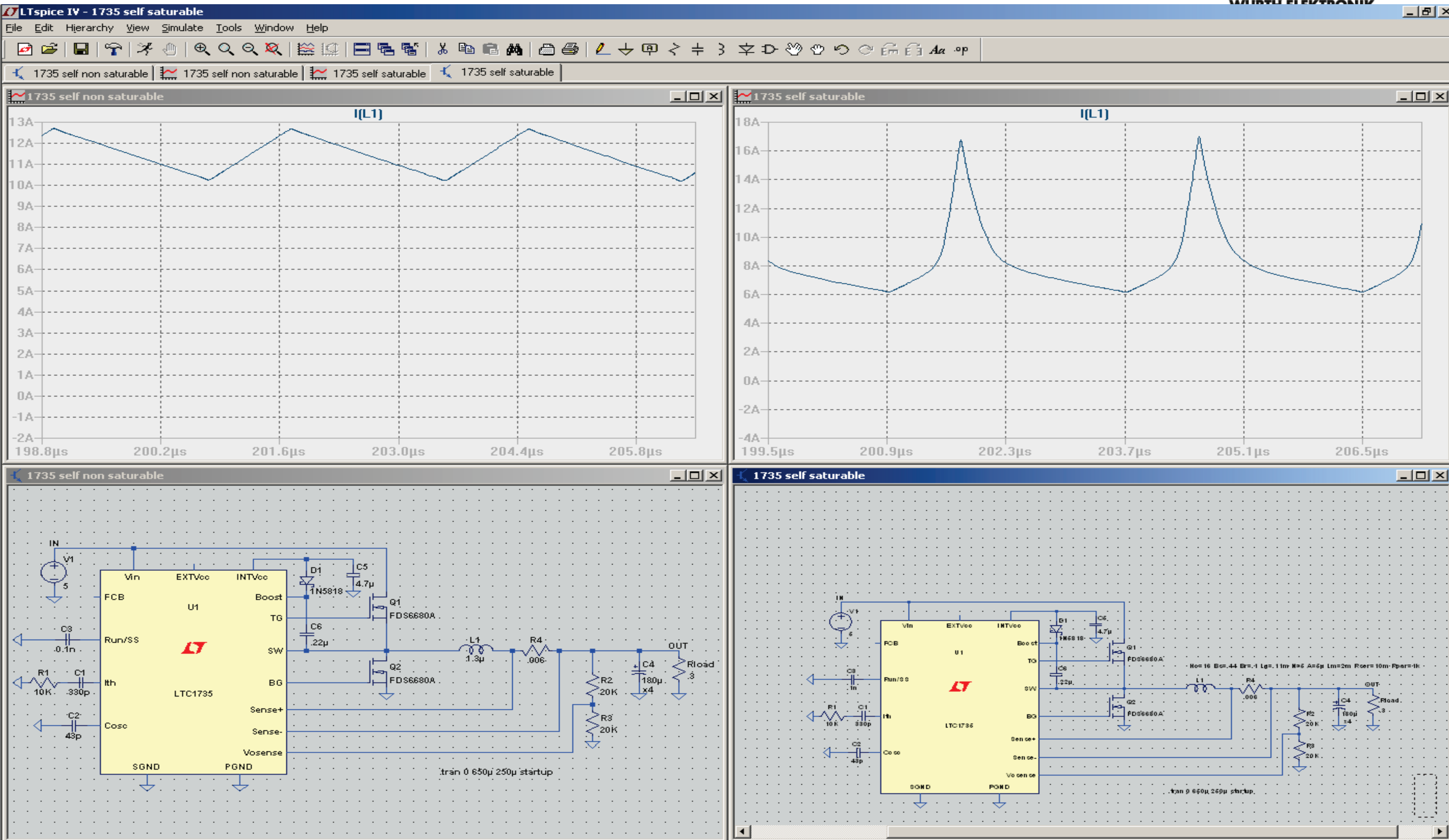
Würth Elektronik:

e.g. WE-PD



- the saturation current always refers to a certain inductance drop and is individually

Saturation current



Inductor selection: How to improve?

- Energy stored

$$Esat := \frac{1}{2}\cdot Lwe\cdot .9\cdot Isat^{2} = 2.559\times 10^{-5}\,J$$

$$E2 := \frac{1}{2}\cdot L2\cdot Ipeak^{2} = 3.107\times 10^{-5}\,J$$

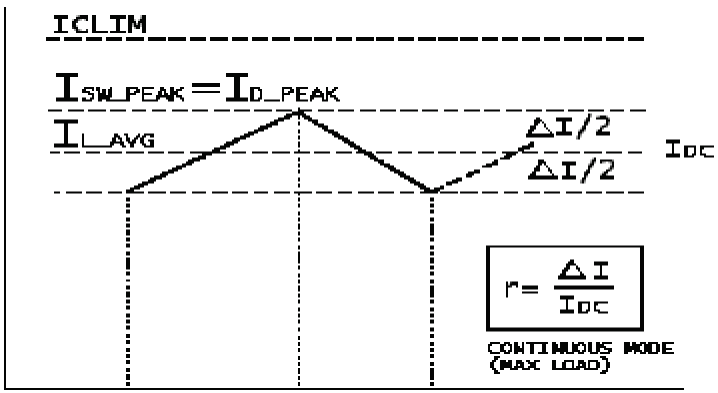
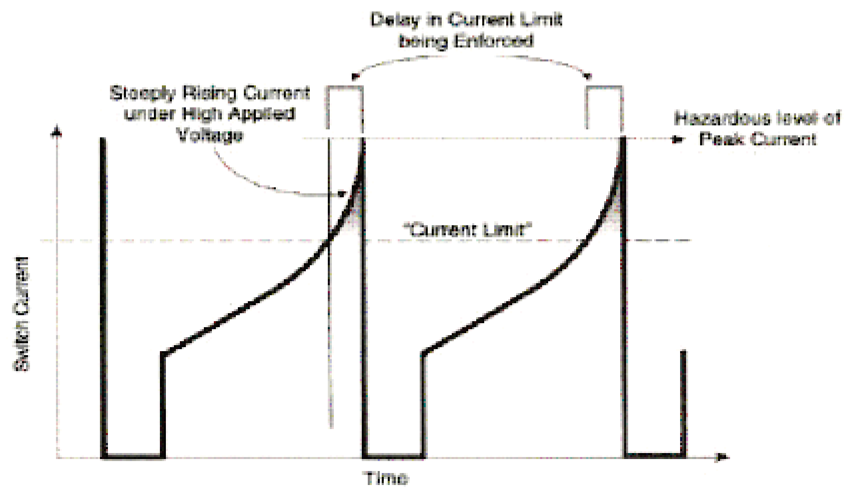
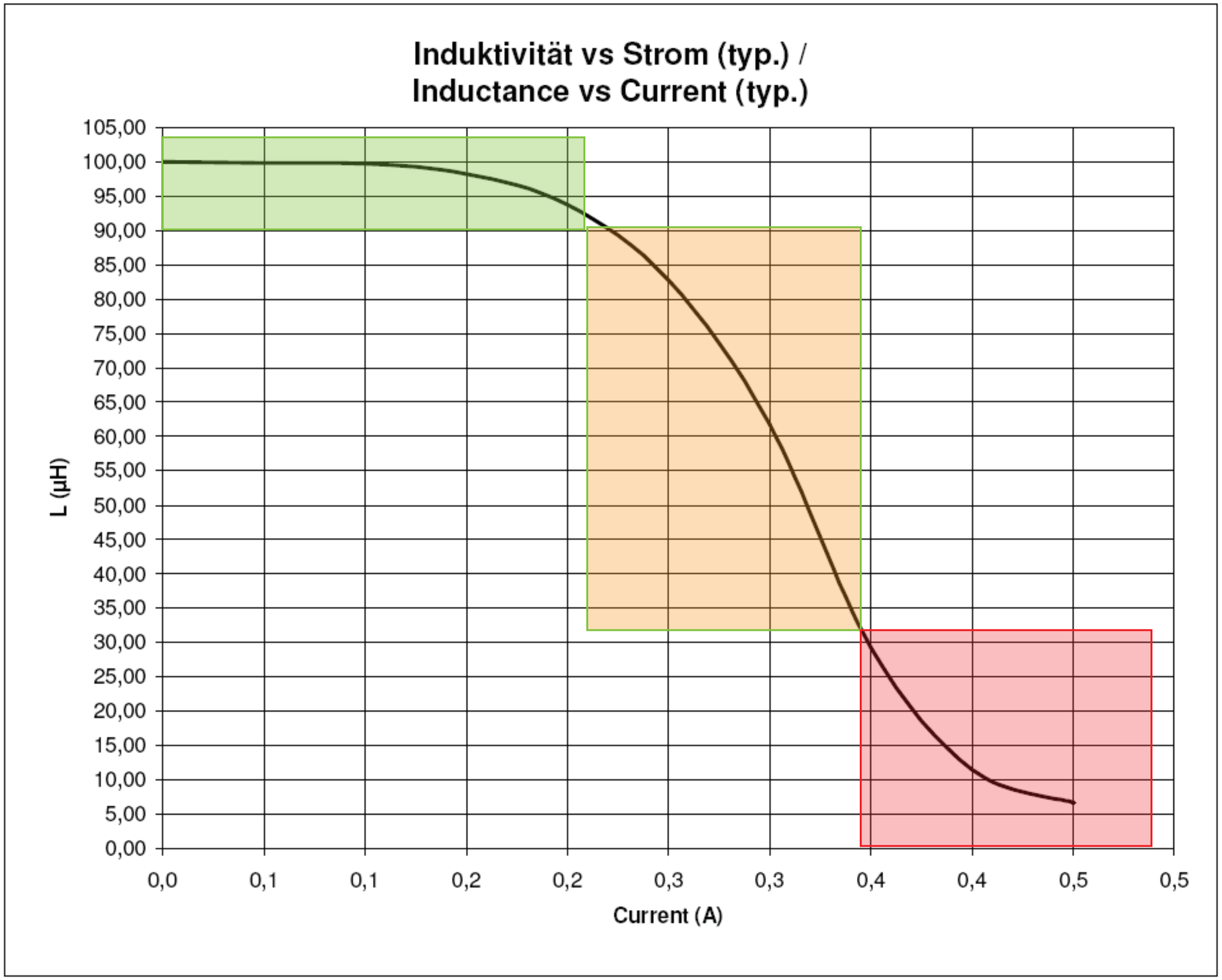


FIGURE 1



Inductor selection: how to improve?



safe operation area

critical operating area

total saturation area

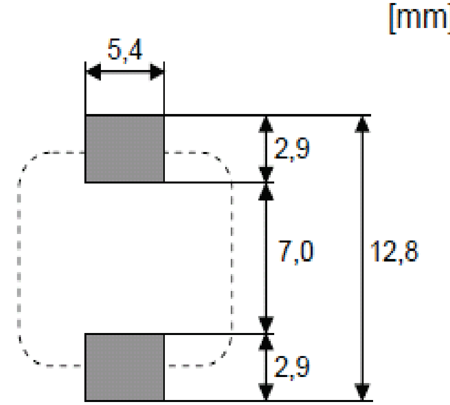
Important when using Inductors made out of Ferrite

Inductor selection: how to improve?

B Elektrische Eigenschaften / Electrical properties :

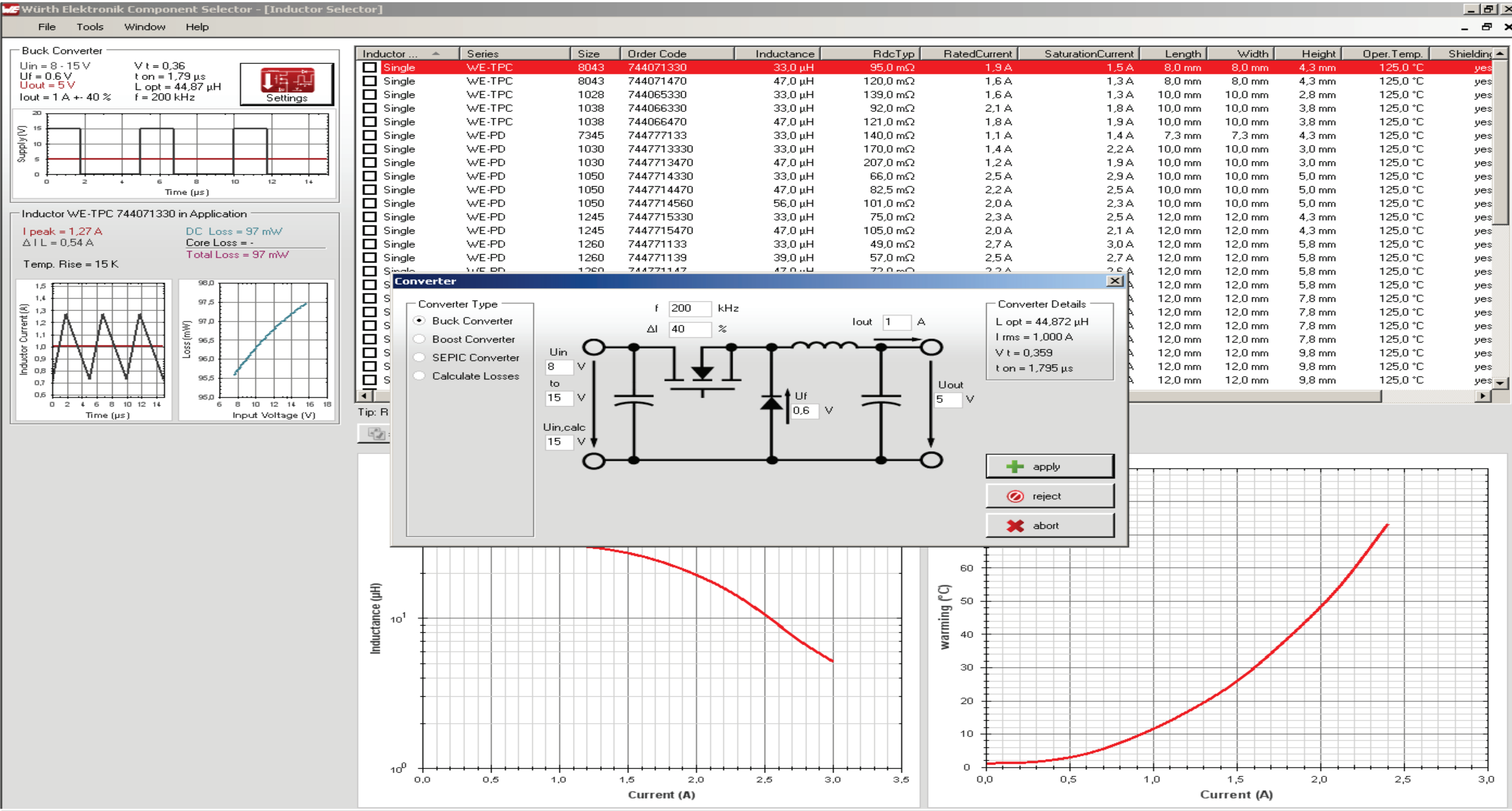
Eigenschaften / properties	Testbedingungen / test conditions		Wert / value	Einheit / unit	tol.
Induktivität / Inductance	**1 kHz / 0,25V**	L	**56,00**	µH	**±20%**
DC-Widerstand / DC-resistance	**@ 20°C**	$R_{DC\ typ.}$	**0,087**	Ω	**typ.**
DC-Widerstand / DC-resistance	**@ 20°C**	$R_{DC\ max.}$	**0,110**	Ω	**max.**
Nennstrom / Rated current	**ΔT=40 K**	I_{DC}	**2,01**	A	**max.**
Sättigungsstrom / Saturation current	**|ΔL/L|<10%**	I_{sat}	**2,35**	A	**typ.**
Eigenres.-Frequenz / Self-res.-frequency		SRF	**7,00**	MHz	**min.**

C Lötpad / Soldering spec. :

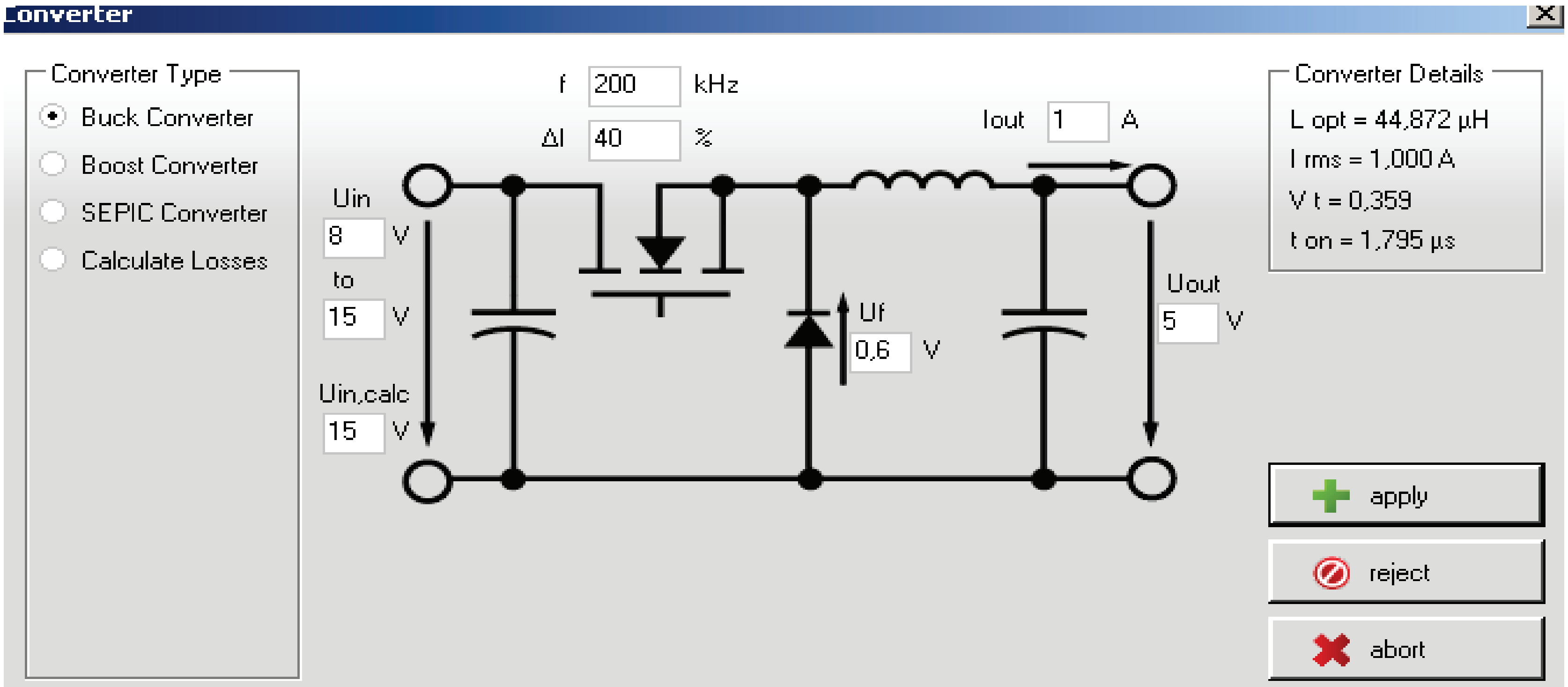


$$Esat := \frac{1}{2} \cdot Lwe \cdot .9 \cdot Isat^2 = 1.392 \times 10^{-4} J$$

Component selector



Component selector



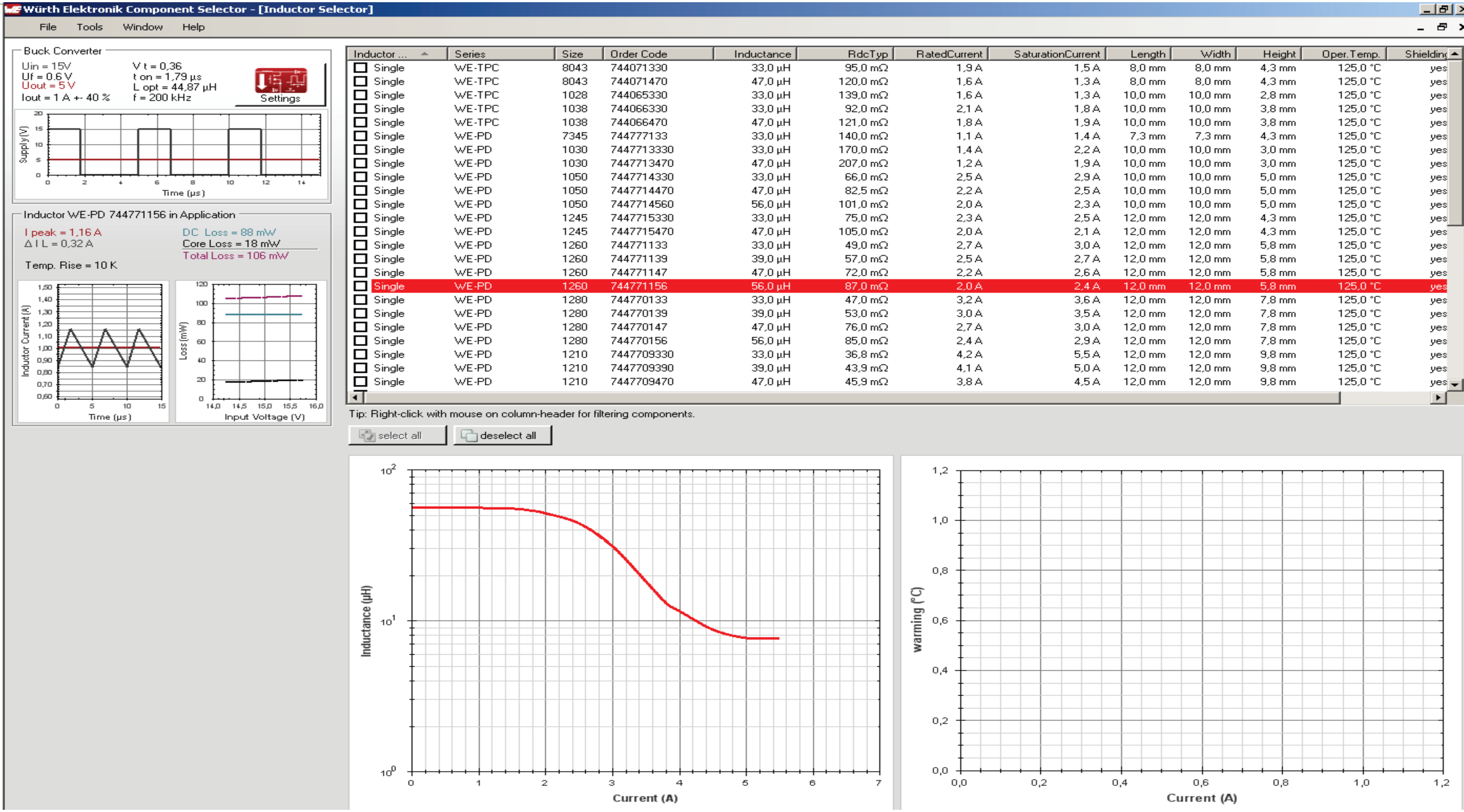
$$\mathrm{Ton4} := \frac{\mathrm{D4}}{\mathrm{Fsw}} = 1.795\times 10^{-6}\,\mathrm{s}$$

$$\mathrm{D4} := \frac{\mathrm{Vout} + \mathrm{Vd}}{\mathrm{Vin} + \mathrm{Vd}} = 0.359$$

$$\mathrm{L4} := \frac{\mathrm{V\mu sec4}}{r\cdot \mathrm{Iout}} = 4.487\times 10^{-5}\,\mathrm{H}$$

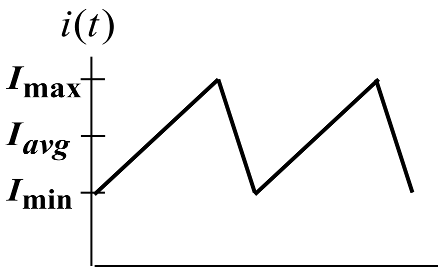
Component selector

WÜRTH ELEKTRONIK



Inductor ...	Series	Size	Order Code	Inductance	RdcTyp	RatedCurrent	SaturationCurrent	Length	Width	Height	Oper.Temp.	Shielding
Single	WE-TPC	8043	744071330	33,0 µH	95,0 mΩ	1,9 A	1,5 A	8,0 mm	8,0 mm	4,3 mm	125,0 °C	yes
Single	WE-TPC	8043	744071470	47,0 µH	120,0 mΩ	1,6 A	1,3 A	8,0 mm	8,0 mm	4,3 mm	125,0 °C	yes
Single	WE-TPC	1028	744065330	33,0 µH	139,0 mΩ	1,6 A	1,3 A	10,0 mm	10,0 mm	2,8 mm	125,0 °C	yes
Single	WE-TPC	1038	744066330	33,0 µH	92,0 mΩ	2,1 A	1,8 A	10,0 mm	10,0 mm	3,8 mm	125,0 °C	yes
Single	WE-TPC	1038	744066470	47,0 µH	121,0 mΩ	1,8 A	1,9 A	10,0 mm	10,0 mm	3,8 mm	125,0 °C	yes
Single	WE-PD	7345	744777133	33,0 µH	140,0 mΩ	1,1 A	1,4 A	7,3 mm	7,3 mm	4,3 mm	125,0 °C	yes
Single	WE-PD	1030	7447713330	33,0 µH	170,0 mΩ	1,4 A	2,2 A	10,0 mm	10,0 mm	3,0 mm	125,0 °C	yes
Single	WE-PD	1030	7447713470	47,0 µH	207,0 mΩ	1,2 A	1,9 A	10,0 mm	10,0 mm	3,0 mm	125,0 °C	yes
Single	WE-PD	1050	7447714330	33,0 µH	66,0 mΩ	2,5 A	2,9 A	10,0 mm	10,0 mm	5,0 mm	125,0 °C	yes
Single	WE-PD	1050	7447714470	47,0 µH	82,5 mΩ	2,2 A	2,5 A	10,0 mm	10,0 mm	5,0 mm	125,0 °C	yes
Single	WE-PD	1050	7447714560	56,0 µH	101,0 mΩ	2,0 A	2,3 A	10,0 mm	10,0 mm	5,0 mm	125,0 °C	yes
Single	WE-PD	1245	7447715330	33,0 µH	75,0 mΩ	2,3 A	2,5 A	12,0 mm	12,0 mm	4,3 mm	125,0 °C	yes
Single	WE-PD	1245	7447715470	47,0 µH	105,0 mΩ	2,0 A	2,1 A	12,0 mm	12,0 mm	4,3 mm	125,0 °C	yes
Single	WE-PD	1260	744771133	33,0 µH	49,0 mΩ	2,7 A	3,0 A	12,0 mm	12,0 mm	5,8 mm	125,0 °C	yes
Single	WE-PD	1260	744771139	39,0 µH	57,0 mΩ	2,5 A	2,7 A	12,0 mm	12,0 mm	5,8 mm	125,0 °C	yes
Single	WE-PD	1260	744771147	47,0 µH	72,0 mΩ	2,2 A	2,6 A	12,0 mm	12,0 mm	5,8 mm	125,0 °C	yes
Single	WE-PD	1260	744771156	56,0 µH	87,0 mΩ	2,0 A	2,4 A	12,0 mm	12,0 mm	5,8 mm	125,0 °C	yes
Single	WE-PD	1280	744770133	33,0 µH	47,0 mΩ	3,2 A	3,6 A	12,0 mm	12,0 mm	7,8 mm	125,0 °C	yes
Single	WE-PD	1280	744770139	39,0 µH	53,0 mΩ	3,0 A	3,5 A	12,0 mm	12,0 mm	7,8 mm	125,0 °C	yes
Single	WE-PD	1280	744770147	47,0 µH	76,0 mΩ	2,7 A	3,0 A	12,0 mm	12,0 mm	7,8 mm	125,0 °C	yes
Single	WE-PD	1280	744770156	56,0 µH	85,0 mΩ	2,4 A	2,9 A	12,0 mm	12,0 mm	7,8 mm	125,0 °C	yes
Single	WE-PD	1210	7447709330	33,0 µH	36,8 mΩ	4,2 A	5,5 A	12,0 mm	12,0 mm	9,8 mm	125,0 °C	yes
Single	WE-PD	1210	7447709390	39,0 µH	43,9 mΩ	4,1 A	5,0 A	12,0 mm	12,0 mm	9,8 mm	125,0 °C	yes
Single	WE-PD	1210	7447709470	47,0 µH	45,9 mΩ	3,8 A	4,5 A	12,0 mm	12,0 mm	9,8 mm	125,0 °C	yes

RMS of common periodic waveforms



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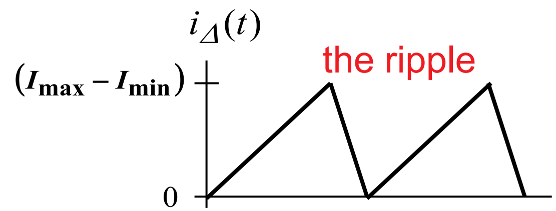
$$I_{avg} = \frac{(I_{max} + I_{min})}{2}$$



+

$$I_{rms}^2 = \frac{(I_{max} - I_{min})^2}{3} + 2I_{min} \bullet \frac{(I_{max} - I_{min})}{2} + I_{min}^2$$

Define $I_{PP} = I_{max} - I_{min}$

$$I_{rms}^2 = \frac{I_{PP}^2}{3} + I_{min} I_{PP} + I_{min}^2$$

RMS of common periodic waveforms

Recognize that $I_{\min} = I_{avg} - \dfrac{I_{PP}}{2}$

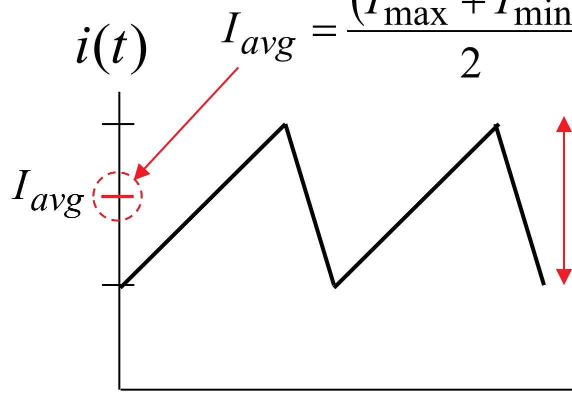
$$I_{rms}^2 = \frac{I_{PP}^2}{3} + \left(I_{avg} - \frac{I_{PP}}{2}\right) I_{PP} + \left(I_{avg} - \frac{I_{PP}}{2}\right)^2$$

$$I_{rms}^2 = \frac{I_{PP}^2}{3} + I_{avg} I_{PP} - \frac{I_{PP}^2}{2} + I_{avg}^2 - I_{avg} I_{PP} + \frac{I_{PP}^2}{4}$$

$$I_{rms}^2 = \frac{I_{PP}^2}{3} - \frac{I_{PP}^2}{4} + I_{avg}^2$$

$$\boxed{I_{rms}^2 = I_{avg}^2 + \frac{I_{PP}^2}{12}}$$

$$I_{avg} = \frac{(I_{\max} + I_{\min})}{2}$$



$$I_{PP} = I_{\max} - I_{\min}$$

The End

- **Thank you**